An Engineering Data Book

Edited by
A J Munday and R A Farrar

REASON FOR PUBLICATION

This book has been produced to provide a pocketable source of data for students pursuing most Engineering Degree Courses, and for use in examinations. * It was not designed for use in Electrical Degree Courses.

It differs from other data books in two respects; it has a comprehensive key-word index and a symbols index in order that users may find data efficiently.

A Professional Engineer should not rely on the memory of facts for use in a design situation, until their frequent use has committed them permanently and accurately to the memory. Until that happy time is reached a data book makes life easier, and makes the permanent retention of accurate facts more likely.

The editors hope that no errors exist but cannot guarantee the accuracy of the data. If you find any errors the editors would appreciate your comments for inclusion in further editions.

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Acknowledgements

i) International Nickel Ltd for the Periodic Table
ii) Biometrika Trustees for the Statistical Tables
iii) Mrs E.H. Day for the layout and typing of the booklet
iv) Members of University Staff who made contributions:- DJC, JRC, RJF, PWF, SPH, RMH, PJL, SJM, JAO, HJS, KVHS, MTT, PLT, DRW, JBW, PW.
v) Other engineering data sources too numerous to mention individually for commonly used values and equations.

* *Where permitted by the examining body.*

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Published by
MACMILLAN EDUCATION LTD
Basingstoke and London

ISBN 0-333-25829-0

Printed in Hong Kong

First edition 1979
Reprinted 1979, 1980, 1982, 1986, 1989

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1. UNITS AND ABBREVIATIONS

1.1 Decimal prefixes

symbol	prefix	factor by which unit is multiplied
T	tera	10^{12}
G	giga	10^{9}
M	mega	10^{6}
k	kilo	10^{3}
h	hecto	10^{2}
da	deca	10
d	deci	10^{-1}
c	centi	10^{-2}
m	milli	10^{-3}
μ	micro	10^{-6}
n	nano	10^{-9}
p	pico	10^{-12}

1.2 SI units

(i) Basic units

unit symbol	unit	quantity
m	metre	length
kg	kilogramme	mass
s	second	time
A	ampere	electric current
K	kelvin	thermodynamic temperature
cd	candela	luminous intensity

(ii) Supplementary and derived units

quantity	unit	symbol	equivalent
plane angle	radian	rad	-
force	newton	N	$kg\ m/s^2$
work, energy heat	joule	J	N m
power	watt	W	J/s
frequency	hertz	Hz	s^{-1}
viscosity:			
kinematic		m^2/s	10^6 cSt (centistoke)
dynamic		Ns/m^2=Pa s	10^3 cP (centipoise)
pressure		Pa=N/m^2	Called pascal,Pa
stress		Pa or N/m^2	
	electrical units		
potential	volt	V	W/A
resistance	ohm	Ω	V/A
charge	coulomb	C	A s
capacitance	farad	F	A s/V
electric field strength	-	V/m	
electric flux density	-	C/m^2	
	magnetic units		
magnetic flux	weber	Wb	V s = Nm/A
inductance	henry	H	V s/A = Nm/A^2
magnetic field strength	-	A/m	
magnetic flux density	tesla	T	Wb/m^2 = N/(Am)

1.3 Conversion factors for other units into SI units

Length, area, volume

1 in = 25.4 mm exactly   1 Å = 10^{-10} m

1 ft = 0.3048 m   1 thou=1 mil = 0.001 in = 25.4 µm

1 yd = 0.914 m   1 micron = 1µm

1 mile = 5280 ft = 1.609 km

1 acre = 0.4047 ha (Hectare) = 4047 m^2

1 in^3 = 16.39 cm^3

1 ft^3 = 0.02832 m^3

1 gal = 0.1605 ft^3 = 4546 cm^3 = 4.546 ℓ (Litre)

1 USgal = 0.1337 ft^3 = 3785 cm^3

Velocity

1 mile/h = 1.467 ft/s = 1.609 km/h = 0.447 m/s

1 knot = 1.689 ft/s = 1.853 km/h = 0.514 m/s

Mass

1 lb = 0.4536 kg

1 slug = 32.17 lb = 14.59 kg

1 ton = 2240 lb = 1016 kg

1 tonne = 1 Mg = 1 metric ton

Flowrate

1 ft^3/s (1 cusec) = 0.02832 m^3/s

1 gal/min = 7.577 10^{-5} m^3/s = 0.07577 dm^3/s

Density

1 lb/in^3 = 27.68 g/cm^3

1 lb/ft^3 = 16.02 kg/m^3

1 slug/ft^3 = 515.4 kg/m^3

Thermal conductivity

1 Btu/ft h deg R = 1.731 J/m s °C = 1.731 W/(mK)

1 cal/cm s deg K = 418.7 J/m s °C = 418.7 W/(mK)

Force

1 pdl = 0.1383 N

1 lbf = 32.17 pdl = 4.448 N

1 tonf = 9964 N

1 kgf = 2.205 lbf = 9.807 N

1 dyne = 10^{-5} N

Torque

1 lbf ft = 1.356 Nm

1 tonf ft = 3037 Nm

Power

1 hp = 550 ft lbf/s = 0.7457 kW

1 ft lbf/s = 1.356 W

1 metric horsepower (ch, PS) = 0.7355 kW

Energy, work, heat

1 ft lbf = 1.356 J

1 kW h = 3.6 MJ

1 Btu = 778.2 ft lbf = 252 cal = 1055 J

1 cal = 4.187 J

1 hp h = 2.685 MJ

Pressure, stress

1 lbf/in^2 = 0.07031 kgf/cm^2 = 6895 N/m^2

1 tonf/in^2 = 157.5 kgf/cm^2 = 15.44 MN/m^2

1 kgf/cm^2 = 0.09807 MN/m^2 = 0.9807 bar

1 kgf/mm^2 = 9.807 MN/m^2 = 0.9807 hbar

1 lbf/ft^2 = 47.88 N/m^2

1 ft H_2O = 62.43 lbf/ft^2 = 2989 N/m^2

1 in Hg = 70.73 lbf/ft^2 = 3386 N/m^2

1 mm Hg = 1 torr = 133.3 N/m^2

1 bar = 14.50 lbf/in^2 = 10^5 N/m^2

1 Int atm = 14.70 lbf/in^2 = 10.34 m water = 1.013 x 10^5 N/m^2
= 1.013 bar = 760 mm Hg = 101.3 kPa

Dynamic viscosity

1 poise (g/cm s)	=	0.1 kg/m s = 0.1 N s/m^2 = 0.1 Pa s
1 kgf s/m^2	=	0.9807 N s/m^2
1 lb/ft h	=	0.4132 mN s/m^2
1 slug/ft s	=	1 lbf s/ft^2 = 47.88 N s/m^2
1 lbf s/in^2	=	6895 N s/m^2

Kinematic viscosity

1 ft^2/s	=	0.09290 m^2/s
1 in^2/s	=	645.2 mm^2/s
1 cSt	=	1 mm^2/s

Electrical units

The conversion factors which follow are from the C.G.S. system to the SI system. (Note: in the C.G.S. system 1 e.m.u. = 3 x 10^{10} e.s.u. of charge).

capacitance	1 e.s.u.	= $\frac{1}{9}$ x 10^{-11} F
charge	1 e.m.u.	= 10 C
current	1 e.m.u.	= 10 A
electric field strength	1 e.s.u.	= 3 x 10^4 V/m
electric flux density	1 e.s.u.	= $\frac{1}{12\pi}$ x 10^{-5} C/m^2
electric polarisation	1 e.s.u.	= $\frac{1}{3}$ x 10^{-5} C/m^2
inductance	1 e.m.u.	= 10^{-9} H
intensity of magnetisation	1 e.m.u.	= 10^3 A/m
magnetic field strength	1 e.m.u.	= $\frac{1}{4\pi}$ x 10^3 A/m
magnetic flux	1 e.m.u.	= 10^{-8} Wb
magnetic flux density	1 e.m.u.	= 10^{-4} Wb/m^2
magnetic moment	1 e.m.u.	= 10^{-3} A m^2
magnetomotive force	1 e.m.u.	= $\frac{10}{4\pi}$ A
mass susceptibility	1 e.m.u/g	= 4π x 10^{-3} kg^{-1}
potential	1 e.m.u.	= 10^{-8} V
resistance	1 e.m.u.	= 10^{-9} Ω

2. PHYSICAL CONSTANTS

Avogadro's number	N	= 6.023 x 10^{26}/(kg mol)
Bohr magneton	β	= 9.27 x 10^{-24} A m^2
Boltzmann's constant	k	= 1.380 x 10^{-23} J/K
Stefan-Boltzmann constant	σ	= 5.67 x 10^{-8} W/(m^2K^4)
characteristic impedance of free space	Z_o	= $(\mu_o/\varepsilon_o)^{\frac{1}{2}}$ = 120πΩ
electron volt	eV	= 1.602 x 10^{-19} J
electron charge	e	= 1.602 x 10^{-19} C
electronic rest mass	m_e	= 9.109 x 10^{-31} kg
electronic charge to mass ratio	e/m_e	= 1.759 x 10^{11} C/kg
Faraday constant	F	= 9.65 x 10^7 C/(kg mol)
permeability of free space	μ_o	= 4π x 10^{-7} H/m
permittivity of free space	ε_o	= 8.85 x 10^{-12} F/m
Planck's constant	h	= 6.626 x 10^{-34} J s
proton mass	m_p	= 1.672 x 10^{-27} kg
proton to electron mass ratio	m_p/m_e	= 1836.1
standard gravitational acceleration	g	= 9.80665 m/s^2 = 9.80665 N/kg
universal constant of gravitation	G	= 6.67 x 10^{-11} N m^2/kg^2
universal gas constant	R_o	= 8.314 kJ/(kg mol K)
velocity of light in vacuo	c	= 2.9979 x 10^8 m/s
volume of 1 kg mol of ideal gas at 1 atm, 0^oC		= 22.41 m^3

Temperature

$$^oC = \frac{5}{9}\ (^oF - 32)$$

$$K = \frac{5}{9}\ (^oF + 459.67) = \frac{5}{9}\ ^oR = \ ^oC + 273.15$$

3 . SUMMARY OF "BASIC"

This language contains the facilities provided in most versions of extended BASIC. Some instructions may vary somewhat from one system to another; however, equivalents should be available. This applies particularly to String Functions, Commands and Control Codes, and to items marked with a †. We suggest that you modify the SYSTEM DEPENDENT INSTRUCTIONS MARKED BY † to conform to your own system and add other instructions in the spaces provided.

Arithmetic Variable Names

numeric variables: e.g. A,X;B4,Z1

arithmetic array variables: e.g. S(4),A(I+1),N2(1,J), C(1,B(1))

String Variable Names

character string variables: e.g. B$

character string array variables: e.g. Z$(4),N$(A,B)

N.B. $ may be £ on some terminals. Use the key 'shift 4'.

Arithmetic Operators

↑	exponentiation e.g. 2↑3 gives 8
-	unary minus
* /	multiplication, division
+ -	addition, subtraction

Operations inside any given pair of brackets are performed before those outside. Subject to this, BASIC performs operations in the order of the operators above. The only(†) exception is A↑-B, interpreted as A↑(-B). Operators of equal priority are applied from left to right.

e.g. 2↑(1+3/2*(1+1)) gives 16

Relational Operators (operate upon arithmetic and string values)

=	>
<	>=
<= (less than equal to)	<> (not equal to)

Logical Operators

AND

XOR exclusive

OR inclusive

Matrix Operators

+ -	addition or subtraction of matrices of equal dimensions
*	multiplication of conformable matrices
*	multiplication of a matrix by a scalar
	e.g. MAT A = (K)*A

Arithmetic Functions (x represents any expression)

PI	has the constant value 3.1415927
SIN(x),COS(x),TAN(x)	sine, cosine, tangent (x in radians)
ATN(x)	arctan (radians)
LOG(x),LOG10(x)	natural log, common log
EXP(x)	exponentiation e↑x where e = 2.71828
SQR(x)	square root
SGN(x)	sign of x (+ve gives 1, 0 gives 0, -ve gives -1)
ABS(x)	absolute value of x (|x|)
INT(x)	largest integer < = x
RND or RND(x)	returns a random number between 0 and 1. x, if present, is ignored.

† String Functions

LEN(A$)	returns the number of characters in the string A$, including trailing blanks
SUB$(A$,N1,N2)	creates a sub string from the string A$ starting with the N1th character and N2 characters long
SUB$(A$,N)	creates a sub string from the string A$, starting with the Nth character to the last character in A$
*CHR$(x)	returns a one character string having the ASCII value x
*ASCII(A$)	returns the ASCII value of the first character in A$
NUM$(N)	creates the string of characters that would be printed by PRINT N;
NUM$(N,field)	creates the string of characters that would be printed by PRINTUSING"field",N;
VAL(A$)	computes the value that would be generated by the INPUT of the characters of A$ to an arithmetic variable

*The ASCII value is based on seven bit characters. Treatment of the parity bit is system dependent.

†Error functions (only valid in an error handling routine entered by ONERROR)

ERR	contains the error number of the most recent error
ERL	contains the line number of the most recent error

Matrix functions

MAT Y = TRN(X)	Y becomes the transpose of X
MAT Y = INV(X)	Y becomes the inverse of X
DET	contains the determinant of X after the evaluation of INV(X)

User defined functions - see DEF statement

Statements

Note - a program line may contain several statements separated by the colon (:) character

Type	Example
CLOSE	CLOSE 2
DATA	DATA 4.3,85,"MONDAY"
DEF	DEF FNA(X) = X↑X DEF FNA1(A,B) = SQR(A+B)
†	DEF FNF(M) IF M = 1 THEN FNF = 1 ELSE FNF = M*FNF(M-1) FNEND
DIM	DIM A(10),B$(5,10)
END	END must be the last statement of a program
FOR	FOR X = 1 TO 10 FOR N = A TO A+R FOR 1 = 2 to 40 STEP 2
GOSUB	GOSUB 200
GOTO	GOTO 151
IF	IF B = A THEN 21 IF A > Z THEN PRINT "BIGGER" IF R < N+1 THEN R = N ELSE R = N+2 IF A > B OR B < C THEN STOP IF FNA(R) = B GOTO 200
INPUT	INPUT A INPUT "TYPE YOUR NAME",A$ INPUT # 4,N,M
LET	LET A = 20 LET A,B,C = 0 A$ = "TEXT" (LET is optional)

MAT	MAT C = CON all elements of C = 1 MAT B = IDN(10,10) identity matrix MAT A = ZER all elements of A = 0 MAT B = ZER(5,10) redimensions and zeros B
MATINPUT	MATINPUT A,B,C(4) MATINPUT # 3,A,C
MATPRINT	MATPRINT B MATPRINT B(10,5); MATPRINT # 2,A
MATREAD	MATREAD A,B(4,4)
NEXT	NEXT I
ON ERROR GOTO	ON ERROR GOTO 140
ON GOSUB	ON X GOSUB 200,250,300 ON FNA(A) + FNB(A) GOSUB 10,15,30,5
ON GOTO	ON A + 1 GOTO 14,25,50
OPEN	†
PRINT	PRINT A,B PRINT "RESULT"; X1 PRINT # 4, I*A,."EXPERIMENT"; N
† PRINT USING	PRINT USING "# # #", A,B PRINT # 3, USING B$,C,Z$ PRINT USING 1000,X
RANDOM	RANDOM
READ	READ A,B$,F1,C
REM	REMARK THIS IS A COMMENT
!	An exclamation mark at the beginning of a line is equivalent to REM An exclamation mark after any statement causes the rest of the line to be treated as comment X = 0! ZERO CONTROL
RESTORE	RESTORE
RESUME	RESUME RESUME 240
RETURN	RETURN
STOP	STOP
TRACE	TRACE
† %	PRINTUSING image, e.g. 1000% X = # #.#

†Commands

RUN	runs the current program
LOAD	loads a program from paper tape (or other medium)
CLEAR NEW	remove any existing program
LIST	LIST prints the current program LIST n prints line n LIST n-m prints lines n to m
DELETE	DELETE 40-45 deletes specified lines from the current program
SAVE	saves a program on paper tape (or other medium)
REP	edits a program line. Any non-numerical character can be used as separator e.g. REP10/X1/A REP30/,B/
RESEQUENCE	renumbers part or all of a program (including GOTO etc references) e.g. RESEQUENCE whole program, steps of 10 RESEQUENCE 900,1000 after old line 900, which becomes 1000, steps of 10 RESEQUENCE,,5 whole program, steps of 5

†Special control codes

ESC ACCEPT	breaks into the program and stops it, typing BASIC READY
CR RETURN	terminate a line of input
\ or $	(on the same key as L, not 4) Abandon the current line of input
← or RUBOUT	delete the previous character or space (may be used repeatedly)

4. ANALYSIS

4.1 Vector algebra

$$\hat{\underline{a}} = \underline{a}/|\underline{a}|$$

$$\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k} \equiv (a_1, a_2, a_3)$$

$$a \equiv |\underline{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$\underline{a} + \underline{b} = (a_1+b_1,\ a_2+b_2,\ a_3+b_3)$$

Scalar (dot) product:

$$\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3 = ab\cos\theta$$

Vector (cross) product:

$$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = ab\sin\theta\,\hat{\underline{n}} \quad \text{where } \hat{\underline{n}} \perp \underline{a},\ \hat{\underline{n}} \perp \underline{b}$$

Triple scalar product:

$$[\underline{a}\ \underline{b}\ \underline{c}] = \underline{a} \cdot \underline{b} \times \underline{c} = \underline{a} \times \underline{b} \cdot \underline{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Triple vector product:

$$\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a}.\underline{c})\underline{b} - (\underline{a}.\underline{b})\underline{c}$$
$$(\underline{a} \times \underline{b}) \times \underline{c} = (\underline{a}.\underline{c})\underline{b} - (\underline{b}.\underline{c})\underline{a}$$

Differentiation of vectors:

$$\frac{d}{dt}(\underline{a} + \underline{b}) = \frac{d\underline{a}}{dt} + \frac{d\underline{b}}{dt} \qquad \frac{d}{dt}(f\,\underline{a}) = \frac{df}{dt}\underline{a} + f\frac{d\underline{a}}{dt}$$

$$\frac{d}{dt}(\underline{a}.\underline{b}) = \underline{a}.\frac{d\underline{b}}{dt} + \frac{d\underline{a}}{dt}.\underline{b} \qquad \frac{d}{dt}(\underline{a} \times \underline{b}) = \underline{a} \times \frac{d\underline{b}}{dt} + \frac{d\underline{a}}{dt} \times \underline{b}$$

$$\frac{d}{dt}(\underline{a}.\underline{b} \times \underline{c}) = \frac{d\underline{a}}{dt}.\underline{b} \times \underline{c} + \underline{a}.\frac{d\underline{b}}{dt} \times \underline{c} + \underline{a}.\underline{b} \times \frac{d\underline{c}}{dt}$$

Gradient:

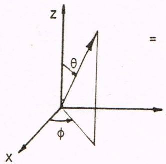
$$\text{grad } V = \nabla V = \underline{i}\frac{\partial V}{\partial x} + \underline{j}\frac{\partial V}{\partial y} + \underline{k}\frac{\partial V}{\partial z} \quad \text{(Cartesian)}$$

$$= \underline{u}_r\frac{\partial V}{\partial r} + \underline{u}_\phi \frac{1}{r}\frac{\partial V}{\partial \phi} + \underline{u}_z\frac{\partial V}{\partial z} \quad \text{(Cylindrical)}$$

where $\underline{u}_r = \underline{i}\cos\phi + \underline{j}\sin\phi$

$\underline{u}_\phi = -\underline{i}\sin\phi + \underline{j}\cos\phi$

$\underline{u}_z = \underline{k}$



$$= \underline{u}_r\frac{\partial V}{\partial r} + \frac{\underline{u}_\theta}{r}\frac{\partial V}{\partial \theta} + \frac{\underline{u}_\phi}{r\sin\theta}\frac{\partial V}{\partial \phi} \quad \text{(Spherical)}$$

where $\underline{u}_r = \underline{i}\cos\phi\sin\theta + \underline{j}\sin\phi\sin\theta + \underline{k}\cos\theta$

$\underline{u}_\theta = \underline{i}\cos\phi\cos\theta + \underline{j}\sin\phi\cos\theta - \underline{k}\sin\theta$

$\underline{u}_\phi = -\underline{i}\sin\phi + \underline{j}\cos\phi$

Divergence:

$$\text{div } \underline{F} = \nabla.\underline{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \quad \text{(Cartesian)}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(rF_r\right) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \quad \text{(Cylindrical)}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2F_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(F_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi} \quad \text{(Spherical)}$$

Curl:

$$\text{curl } \underline{F} = \nabla\text{x}\underline{F} = \underline{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \underline{j}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \underline{k}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right) \quad \text{(Cartesian)}$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \frac{1}{r}\begin{vmatrix} \underline{u}_r & r\underline{u}_\phi & \underline{u}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_r & rF_\phi & F_z \end{vmatrix} \quad \text{(Cylindrical)}$$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \underline{u}_r & r\underline{u}_\theta & r\sin\theta\,\underline{u}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ F_r & rF_\theta & r\sin\theta F_\phi \end{vmatrix} \quad \text{(Spherical)}$$

Laplace:

$$\nabla.\nabla V = \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \quad \text{(Cartesian)}$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} \quad \text{(Cylindrical)}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$

(Spherical)

Space curves:

$\underline{v} = \underline{u}\frac{ds}{dt}$, s = arc length, $\underline{u}$ = unit tangent

$\underline{n}$ = unit 'inward' normal

$\underline{a} = \frac{v^2}{\rho}\underline{n} + \frac{dv}{dt}\underline{u}$

$\frac{d\underline{u}}{ds} = \frac{1}{\rho}\underline{n}$ ρ = radius of curvature

$\underline{b} = \underline{u} \times \underline{n}$, $\underline{b}$ = binormal vector

$\frac{d\underline{b}}{ds} = -\frac{1}{\tau}\underline{n}$, $\frac{d\underline{n}}{ds} = \frac{1}{\tau}\underline{b} - \frac{1}{\rho}\underline{u}$, $\frac{1}{\tau}$ = torsion

Identities:

$\nabla.\phi\underline{u} = \phi\nabla.\underline{u} + \underline{u}.\nabla\phi$

$\nabla\text{x}\phi\underline{u} = \phi\nabla\text{x}\underline{u} + \nabla\phi\text{x }\underline{u}_o$

$\nabla.\underline{u}\text{x}\underline{v} = \underline{v}.\nabla\text{x}\underline{u} - \underline{u}.\nabla\text{x}\underline{v}$

4.2 Series

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \ldots,$$

for arbitrary α, $|x| < 1$

$$e^x = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + \frac{(-1)^n}{(2n)!}x^{2n} + \ldots \quad \text{for all } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + \ldots \quad \text{for all } x$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \ldots \quad \text{for } |x| < \pi/2$$

$$\ell n(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + \frac{(-1)^n}{(n+1)}x^{n+1} + \ldots$$

$$\text{for } \; -1 < x \leqslant 1$$

Taylor's

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \ldots$$

$$+ \frac{h^{n-1}}{(n-1)!}f^{(n-1)}(a) + \frac{h^n}{n!}f^{(n)}(c) \quad \text{where } a < c < a+h$$

Maclaurin's

$$f(x) = f(o) + xf'(o) + \frac{x^2}{2!}f''(o) + \ldots$$

$$+ \frac{x^{n-1}}{(n-1)!}f^{(n-1)}(o) + \frac{x^n}{n!}f^{(n)}(\theta x) \quad \text{where } o < \theta < 1$$

Stirling's formula for n!

For n large, $n! \sim \sqrt{(2\pi)}\, n^{n+\frac{1}{2}} e^{-n}$

or, $\log_{10} n! \approx 0.39909 + (n+\frac{1}{2})\log_{10} n - 0.43429n$.

Fourier series

(i) General formulae

If $f(x)$ is periodic of period 2L, $f(x+2L) = f(x)$

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L}$$

where

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx \qquad n = 0, 1, 2, \ldots$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx \qquad n = 1, 2, 3, \ldots$$

If $f(x)$ is an even function of x, i.e., $f(-x) = f(x)$

$$\text{then } a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx \qquad n = 0, 1, 2 \ldots$$

$$\text{and } b_n = 0 \qquad n = 1, 2, 3, \ldots$$

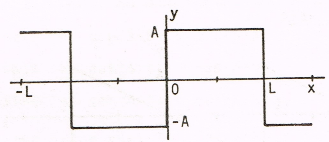
If $f(x)$ is an odd function of x, i.e., $f(-x) = -f(x)$

$$\text{then } a_n = 0 \qquad n = 0, 1, 2, \ldots$$

$$\text{and } b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx \qquad n = 1, 2, 3, \ldots$$

(ii) Special waveforms, all of period 2L

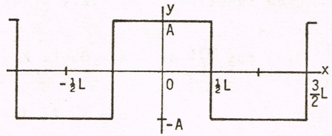
(a) Square wave, sine series



$$f(x) = \frac{4A}{\pi}\left[\sin\frac{\pi x}{L} + \frac{1}{3}\sin\frac{3\pi x}{L} + \frac{1}{5}\sin\frac{5\pi x}{L} + \ldots\right]$$

mean square value $= A^2$

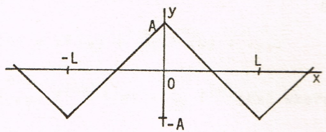
(b) Square wave, cosine series



$$f(x) = \frac{4A}{\pi}\left[\cos\frac{\pi x}{L} - \frac{1}{3}\cos\frac{3\pi x}{L} + \frac{1}{5}\cos\frac{5\pi x}{L} - \ldots\right]$$

mean square value $= A^2$

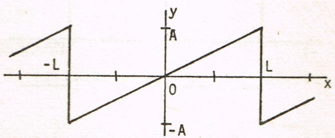
(c) Triangular wave



$$f(x) = \frac{8A}{\pi^2}\left[\cos\frac{\pi x}{L} + \frac{1}{3^2}\cos\frac{3\pi x}{L} + \frac{1}{5^2}\cos\frac{5\pi x}{L} + \ldots\right]$$

mean square value $= \frac{A^2}{3}$

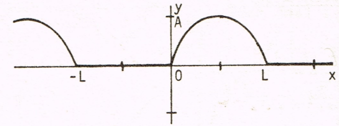
(d) Saw-tooth wave



$$f(x) = \frac{2A}{\pi}\left[\sin\frac{\pi x}{L} - \frac{1}{2}\sin\frac{2\pi x}{L} + \frac{1}{3}\sin\frac{3\pi x}{L} - \ldots\right]$$

mean square value $= \frac{A^2}{3}$

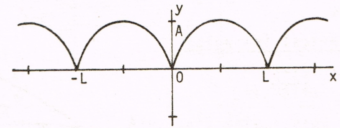
(e) Half-wave rectification



$$f(x) = \frac{A}{2}\sin\frac{\pi x}{L} + \frac{2A}{\pi}\left[\frac{1}{2} - \frac{1}{3}\cos\frac{2\pi x}{L} - \frac{1}{15}\cos\frac{4\pi x}{L} - \ldots\right]$$

mean square value $= \frac{A^2}{4}$ average value $= \frac{A}{\pi}$

(f) Full-wave rectification



$$f(x) = \frac{4A}{\pi}\left[\frac{1}{2} - \frac{1}{3}\cos\frac{2\pi x}{L} - \frac{1}{15}\cos\frac{4\pi x}{L} - \ldots\right]$$

mean square value $= \frac{A^2}{2}$ average value $= \frac{2A}{\pi}$

4.3 Trigonometric, hyperbolic and algebraic relations

$$2\sin A\cos B = \sin(A-B) + \sin(A+B)$$
$$2\cos A\cos B = \cos(A-B) + \cos(A+B)$$
$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

$$\sin A + \sin B = 2\sin\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\sin A - \sin B = 2\cos\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$
$$\cos A + \cos B = 2\cos\tfrac{1}{2}(A+B)\cos\tfrac{1}{2}(A-B)$$
$$\cos A - \cos B = -2\sin\tfrac{1}{2}(A+B)\sin\tfrac{1}{2}(A-B)$$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin^2 A + \cos^2 A = 1$

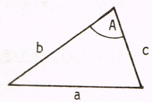
$\text{Sec}^2 A = \text{Tan}^2 A + 1$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

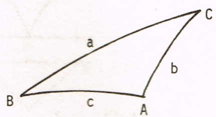
$\sin A = \dfrac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a+b+c)$

$= \dfrac{2}{bc}$ area

$a^2 = b^2 + c^2 - 2\,bc\cos A$



Relation for Spherical Triangles



$\dfrac{\sin a}{\sin A} = \dfrac{\sin b}{\sin B} = \dfrac{\sin c}{\sin C}$

$\cos a = \cos b \cos c + \sin b \sin c \cos A$

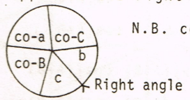
$\cos A = -\cos B \cos C + \sin B \sin C \cos a$

$\sin\frac{A}{2} = \sqrt{\dfrac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$ where $s = \frac{1}{2}(a+b+c)$

$\sin\frac{a}{2} = \sqrt{-\dfrac{\cos S \cos(S-A)}{\sin B \sin C}}$ where $S = \frac{1}{2}(A+B+C)$

Napiers Rules for right spherical triangles:

Arrange the five parts about the right angle with 'co' attached to the three parts opposite the right angle. E.g. for the right angle at A we have

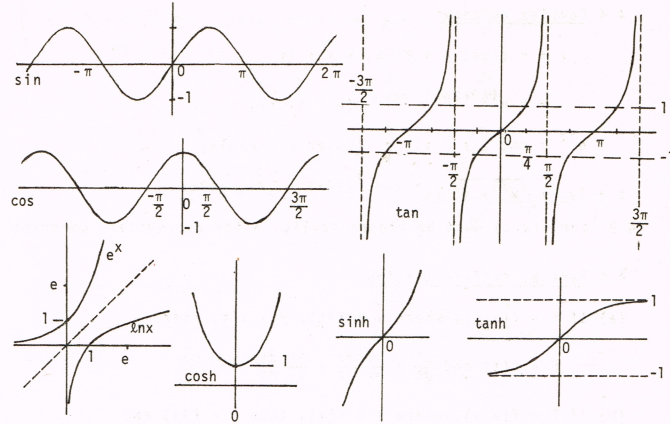


N.B. co-a is the complement of a.

i.e. $90^o - a$

Then: The sine of the middle part is the product of the tangents of adjacent parts and is the product of the cosines of opposite parts.

N.B. A leg and its opposite angle are always in the same quadrant. If the hypotenuse is less than 90^o the legs are in the same quadrant, otherwise they are in opposite quadrants.



$\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$ $\qquad$ $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$

$\sinh x = \dfrac{e^{x} - e^{-x}}{2}$ $\qquad$ $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

$\cos iz = \cosh z$ $\qquad$ $\sin iz = i \sinh z$

$\cosh iz = \cos z$ $\qquad$ $\sinh iz = i \sin z$

$e^z = \cosh z + \sinh z$

$\log_{10}(10^x) \equiv \log_{10}(\text{antilog}_{10} x) \equiv x \equiv 10^{\log_{10} x} \equiv e^{\log_e x} = e^{\ell n x}$

$a^2 - b^2 = (a+b)(a-b)$: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

equations of curves

circle	ellipse	hyperbola	parabola
$x^2 + y^2 = a^2$	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$	$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$	$y^2 = ax$

4.4 Complex numbers

$$z = r\,(\cos\theta + i\sin\theta) = x + iy$$

$$= r\,e^{i(\theta+2n\pi)} \qquad (n = 0, \pm1, \pm2, \ldots)$$

$$e^{iz} = \cos z + i\sin z \quad \text{[Euler's Formula]}$$

$$x + iy = \sqrt{x^2+y^2}\;e^{i\tan^{-1}(y/x)}; \quad z^c = e^{c\,\ell n z}$$

N.B. $\tan^{-1}(y/x)$ must be chosen to lie in the appropriate quadrant

4.5 Partial differentiation

(a) If $F = f(x,y)$, where $x = X(t)$, $y = Y(t)$ then

$$F = F(t) \text{ and } \frac{dF}{dt} = \frac{\partial f}{\partial x}\frac{dX}{dt} + \frac{\partial f}{\partial y}\frac{dY}{dt}$$

(b) If $F = f(x,y)$, where $y = Y(x)$, then $F = F(x)$ and

$$\frac{dF}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dY}{dx}$$

(c) If $F = f(x,y)$, where $x = X(u,v)$, $y = Y(u,v)$ then

$$F = F(u,v) \text{ and } \frac{\partial F}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial X}{\partial u} + \frac{\partial f}{\partial y}\frac{\partial Y}{\partial u},$$

$$\frac{\partial F}{\partial v} = \frac{\partial f}{\partial x}\frac{\partial X}{\partial v} + \frac{\partial f}{\partial y}\frac{\partial Y}{\partial v}.$$

4.6 Differential Equations

(i) First Order

Type	Characteristic	Method of solution
separable	$y' = P(x)Q(y)$	rearrange:- $\int\frac{1}{Q}\,dy = \int P\,dx + c$
homogeneous	$y' = f\left(\frac{y}{x}\right)$	by substitution $y = ux$ to make equation separable
exact	$M(x,y)dx + N(x,y)dy$ where $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	$\frac{\partial G}{\partial x} = M$, $\frac{\partial G}{\partial y} = N$ Solve for G
linear	$y' + P(x)y = Q(x)$	multiply through by $e^{\int P dx}$

(ii) Second Order, linear with constant coefficients

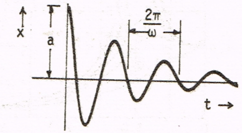
$$m\ddot{x} + \alpha\dot{x} + kx = 0$$

$$\ddot{x} + 2\xi\omega_o\dot{x} + \omega_o^2 x = 0, \quad \xi = \frac{\alpha}{2\sqrt{mk}}, \; \omega_o = \sqrt{\frac{k}{m}}$$

(a) $\xi < 1$ (underdamping)

$$x = a\,e^{-\xi\omega_o t}\cos(\omega t - \theta)$$

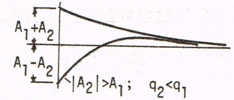
$$\omega = \omega_o\sqrt{1-\xi^2}$$



(b) $\xi > 1$ (overdamping)

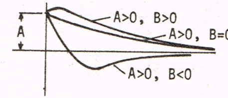
$$x = A_1 e^{-q_1 t} + A_2 e^{-q_2 t}$$

where $q_1, q_2 = \omega_o\,\{\xi \pm \sqrt{(\xi^2-1)}\}$



(c) $\xi = 1$ (critical damping)

$$x = (A + Bt)e^{-\xi\omega_o t}$$



Forced oscillations

$$\ddot{x} + 2\xi\omega_o\dot{x} + \omega_o^2 x = a\cos pt, \quad a = \frac{F}{m}, \; x_1 = \frac{F}{k}$$

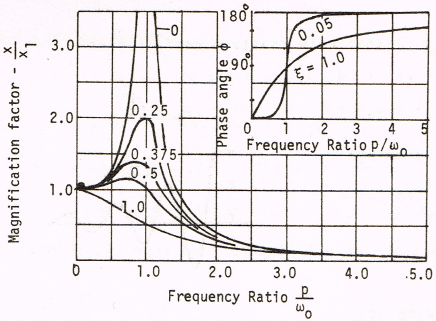
$$x = \frac{Aa}{\omega_o^2}\cos(pt - \phi)$$

$$\tan\phi = \frac{2\,\xi\,p/\omega_o}{1 - \left(\frac{p}{\omega_o}\right)^2}$$

$$A = \left|\frac{x}{x_1}\right| = \frac{1}{\left[\left(1-\left(\frac{p}{\omega_o}\right)^2\right)^2 + \left(2\xi\frac{p}{\omega_o}\right)^2\right]^{\frac{1}{2}}}$$

At resonance $p = \omega_o\sqrt{1 - 2\xi^2}$

$$x = \frac{x_1}{2\xi\sqrt{1-\xi^2}} \qquad \tan\phi = \frac{\sqrt{1-2\xi^2}}{\xi}$$



4.7 Rules of Differentiation and Integration

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw) = uv\frac{dw}{dx} + uw\frac{dv}{dx} + vw\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{1}{v^2}\left(v\frac{du}{dx} - u\frac{dv}{dx}\right)$$

$$\int uv\,dx = uw - \int \frac{du}{dx} w\,dx, \text{ where } w = \int v\,dx$$

4.8 Standard Differentials and Integrals

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \int x^n\,dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$\frac{d}{dx}\ln|x| = \frac{1}{x} \qquad \int \frac{dx}{x} = \ln|x|$$

$$\frac{d}{dx}e^{ax} = ae^{ax} \qquad \int e^{ax}\,dx = \frac{e^x}{a} \quad a \neq 0$$

$$\frac{d}{dx}a^x = a^x \ln a \qquad \int a^x\,dx = \frac{a^x}{\ln a}, \quad a > 0, \quad a \neq 1$$

$$\frac{d}{dx}x^x = x^x(1+\ln x)$$

$$\int \ln x\,dx = x(\ln x - 1)$$

$$\frac{d}{dx}\sin x = \cos x \qquad \int \cos x\,dx = \sin x$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \int \sin x\,dx = -\cos x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \int \sec^2 x\,dx = \tan x$$

$$\frac{d}{dx}\cot x = -\mathrm{cosec}^2 x \qquad \int \mathrm{cosec}^2 x\,dx = -\cot x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{(1-x^2)}} \qquad \int \frac{dx}{\sqrt{(1-x^2)}} = \sin^{-1} x, \quad |x| < 1$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2} \qquad \int \frac{dx}{1+x^2} = \tan^{-1} x$$

$$\frac{d}{dx}\cosh x = \sinh x \qquad \int \sinh x\,dx = \cosh x$$

$$\frac{d}{dx}\sinh x = \cosh x \qquad \int \cosh x\,dx = \sinh x$$

$$\frac{d}{dx}\tanh x = \mathrm{sech}^2 x \qquad \int \mathrm{sech}^2 x\,dx = \tanh x$$

$$\frac{d}{dx}\coth x = -\mathrm{cosech}^2 x \qquad \int \mathrm{cosech}^2 x\,dx = -\coth x$$

$$\frac{d}{dx}\sinh^{-1} x = \frac{1}{\sqrt{(1+x^2)}} \qquad \int \frac{dx}{\sqrt{(1+x^2)}} = \sinh^{-1} x = \ln|x + \sqrt{(1+x^2)}|$$

$$\frac{d}{dx}\cosh^{-1} x = \frac{1}{\sqrt{(x^2-1)}} \qquad \int \frac{dx}{\sqrt{(x^2-1)}} = \cosh^{-1} x = \ln|x + \sqrt{(x^2-1)}|, x \geq 1$$

$$\frac{d}{dx}\tanh^{-1} x = \frac{1}{1-x^2} \qquad \int \frac{dx}{1-x^2} = \tanh^{-1} x = \tfrac{1}{2}\ln\left|\frac{1+x}{1-x}\right|, \quad x^2 < 1$$

$$\frac{d}{dx}\coth^{-1} x = \frac{1}{1-x^2} \qquad \int \frac{dx}{x^2-1} = -\coth^{-1} x = \tfrac{1}{2}\ln\left|\frac{x-1}{x+1}\right|, \quad x^2 > 1$$

Some definite integrals (m,n integers)

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \begin{cases} \frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\frac{3}{4}\,\frac{1}{2}\,\frac{\pi}{2}, & n \text{ even} \\ \frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\frac{4}{5}\,\frac{2}{3}\,1, & n \text{ odd} \end{cases}$$

$$I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x\,dx = \left(\frac{m-1}{m+n}\right) I_{m-2,n} = \left(\frac{n-1}{m+n}\right) I_{m,n-2},\quad m \neq -n$$

$$\int_0^{\pi} \sin mx \,\sin nx\,dx = \int_0^{\pi} \cos mx\,\cos nx\,dx = 0 \quad (m \neq n)$$

$$\int_0^{\pi} \sin nx\,\cos nx\,dx = 0$$

$$\int_0^{\infty} e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2},\ a > 0$$

$$\int_0^{\infty} e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2},\ a > 0$$

$$\int_0^{\infty} e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

The error function $\quad \operatorname{erf} z = \frac{2}{\sqrt{\pi}}\int_0^z e^{-u^2}\,du$ (refer to page 30 for tabulated value)

$$\int_0^{\pi} \frac{\sin\theta\,\cos\theta}{(1+\varepsilon\cos\theta)^3}\,d\theta = \frac{-2\varepsilon}{(1-\varepsilon^2)^2}$$

$$\int_0^{\pi} \frac{\sin^2\theta\,d\theta}{(1+\varepsilon\cos\theta)^3} = \frac{\pi}{2(1-\varepsilon^2)^{3/2}}$$

$$\int_0^{2\pi} \frac{d\theta}{(1+\varepsilon\cos\theta)} = \frac{2\pi}{(1-\varepsilon^2)^{1/2}}$$

4.9 Laplace Transforms

Definition

$$F(s) = L[f(t)] = \int_0^{\infty} f(t)e^{-st}\,dt$$

Theorems

Linearity	$L[af(t)+bg(t)]$	$= aF(s)+bG(s)$
Final Value	$\lim_{t\to\infty} f(t)$	$= \lim_{s\to 0} sF(s)$
Initial Value	$\lim_{t\to 0} f(t)$	$= \lim_{s\to\infty} sF(s)$
Differentiation	$L\left[\frac{df(t)}{dt}\right]$	$= sF(s)-f(0)$
	$L\left[\frac{d^2f(t)}{dt^2}\right]$	$= s^2F(s)-sf(0)-f^1(0)$
Integration	$L\left[\int f(t)dt\right]$	$= \frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$
First Shifting	$L[e^{at}f(t)]$	$= F(s-a)$
Second Shifting	$L[f(t-a)]\quad t>a$	$= e^{-as}F(s)$
Convolution $L[f*g] \equiv$	$L\left[\int_0^t f(u)g(t-u)du\right]$	$= F(s)G(s)$
Partial Differentiation	$L\left[\frac{\partial f(t,\alpha)}{\partial\alpha}\right]$	$= \frac{\partial}{\partial\alpha} F(s,\alpha)$
Time Multiplication	$L[tf(t)]$	$= \frac{-dF(s)}{ds}$

Transform Pairs

Function	Laplace Transform
1	$\frac{1}{s}$
$H(t-T) = 0 \quad t < T$ $ = 1 \quad t \geq T$	$\frac{1}{s}e^{-sT}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$1 - e^{-t}/T$	$\frac{1}{s(1+Ts)}$
$\frac{\omega_n}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}\,t\right)$	$\frac{1}{1+2\frac{\xi s}{\omega_n}+\frac{s^2}{\omega_n^2}}$

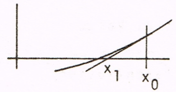
$1 - \frac{1}{\sqrt{1-\xi^2}}\, e^{-\xi\omega_n t} \sin\left(\omega_n\sqrt{1-\xi^2}\,t + \cos^{-1}\xi\right)$ $\qquad$ $\frac{1}{s\left[1 + 2\frac{\xi s}{\omega_n} + \frac{s^2}{\omega_n^2}\right]}$

4.10 Numerical analysis

(i) Approximate solution of an algebraic equation $f(x) = 0$

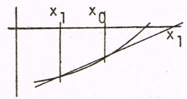
(a) Newton's Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$



(b) Secant Method

$$x_1 = \frac{-x_0\, f(x_{-1}) + x_{-1}\, f(x_0)}{f(x_0) - f(x_{-1})}$$



(ii) Least-squares fitting of a straight line

If y_i $(i = 1, 2, \ldots n)$ are the experimentally observed values of y at chosen (exact) values of x_i of the variable x, the line of 'best fit' passes through the centroid

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

and is given by $y = mx + c$ where,

$$m = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}, \quad c = \bar{y} - m\bar{x}$$

$$= \frac{\Sigma x_i y_i - n\bar{x}\bar{y}}{\Sigma x_i^{\,2} - n\bar{x}^2}$$

(iii) Finite-difference formulae

$$\Delta f(x) = f(x+h) - f(x)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + O(h^2)$$

$$f''(x) = \frac{f(x+h) - 2\,f(x) + f(x-h)}{h^2} + O(h^2)$$

$$f'''(x) = \frac{f(x+2h) - 2\,f(x+h) + 2\,f(x-h) - f(x-2h)}{2h^3}$$

(iv) Lagrange's interpolation formula for unequal intervals.

The polynomial $P(x)$ of degree 2 passing through the three points (x_i, y_i), $i = 1, 2, 3$, is

$$P(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}\, y_1 + \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}\, y_2 + \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}\, y_3$$

(v) Formulae for numerical integration

Equal intervals h

$$x_n = x_0 + nh, \qquad y_n = y(x_n)$$

(a) Trapezoidal Rule (1-strip):

$$\int_{x_0}^{x_1} y(x)\,dx = \frac{h}{2}[y_0 + y_1] + \varepsilon,$$

$$\varepsilon \simeq \frac{-h^3}{12}\, y''_0 \quad \text{or,} \quad \frac{-h}{12}\,\Delta^2 y_0$$

(b) Simpson's Rule (2-strip):

$$\int_{x_0}^{x_2} y(x)\,dx = \frac{h}{3}[y_0 + 4y_1 + y_2] + \varepsilon,$$

$$\varepsilon \simeq \frac{-h^5}{90}\, y_1^{(4)}, \text{ or } \frac{-h}{90}\,\Delta^4 y_0$$

(vi Runge-Kutta

2nd order: $y_{n+1} = y_n + \frac{h}{2}\left\{f(x_n, y_n) + f(x_n+h, y_n+k_1)\right\}$

4th order: $y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_n, y_n)$

$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$k_4 = hf(x_n + h, y_n + k_3)$

5. ANALYSIS OF EXPERIMENTAL DATA

5.1 Probability distributions for discrete random variables

Notation: $P(r) = f(r) \Longrightarrow$ the probability distribution of random variable r is $f(r)$

$$\mu \quad = \text{mean value of } r = \sum_{i=1}^{N} r_i f(r_i)$$

$$\sigma^2 \quad = \text{variance of } r = \sum_{i=1}^{N} r_i^2 f(r_i) - \mu^2$$

$$\binom{n}{r} \quad = \text{binomial coefficient} = \frac{n!}{(n-r)!r!} = \binom{n}{n-r}$$

evaluate using Pascal's Triangle

	r = 0	1	2	3	4	5	6	7	8	9	10
n = 0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1

(a) Binomial:

n = number of trials with constant probability p of success in each

r = number of successes

$$P(r) = \binom{n}{r} p^r (1-p)^{n-r} \qquad r = 0, 1, 2, \dots n$$

$$\mu = np, \qquad \sigma^2 = np(1-p)$$

(b) Poisson:

μ = mean rate of occurrence of an event

r = number of events actually occurring in unit time

$$P(r) = e^{-\mu} \mu^r / r! \qquad r = 0, 1, \dots$$

$$\sigma^2 = \mu$$

5.2 Probability distributions for continuous random variables

(a) Exponential:

probability density function $f(x) = \lambda e^{-\lambda x}$,

$x \geq 0, \ \lambda > 0$

$$\mu = 1/\lambda \qquad \sigma^2 = 1/\lambda^2$$

(b) Normal: the standardised normal distribution, $N(0,1)$

has probability density function

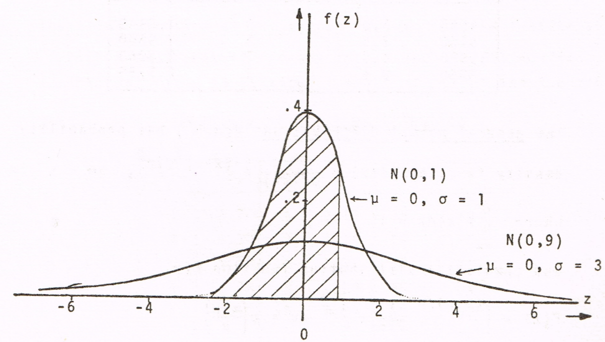
$$\phi(z) = \frac{1}{\sqrt{(2\pi)}} e^{-\frac{1}{2}z^2}$$

$$\mu = 0 \qquad \sigma = 1$$

Φ = cumulative distribution function

$\Phi(z)$ = probability that the random variable is observed to have a value $\leq z$ (the shaded area shown)

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \, dt$$



For negative z use $\Phi(-z) = 1 - \Phi(z)$

z	$\Phi(z)$	z	$\Phi(z)$	z	$\Phi(z)$
0.0	.5000	1.0	.8413	2.0	.9772
.1	.5398	.1	.8643	.1	.9821
.2	.5793	.2	.8849	.2	.9861
.3	.6179	.3	.9032	.3	.9893
.4	.6554	.4	.9192	.4	.9918
0.5	.6915	1.5	.9332	2.5	.9938
.6	.7257	.6	.9452	.6	.9953
.7	.7580	.7	.9554	.7	.9965
.8	.7881	.8	.9641	.8	.9974
.9	.8159	.9	.9713	.9	.9981
				3.0	.9987
				4.0	.99997

Percentage points of the Normal Distribution N(0,1)

$\Phi(z)$	%(1-tail)	%(2-tails)	z
.9500	5.0	10	1.6449
.9750	2.5	5	1.9600
.9900	1.0	2	2.3263
.9950	0.5	1	2.5758

The general normal distribution $N(\mu,\sigma^2)$ has probability

density function $f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$

where $\int_{-\infty}^{\infty} f(x)dx = 1$

and cumulative distribution function F(x)

$$F(x) = \int_{-\infty}^{(x-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2} du = \Phi\left(\frac{x-\mu}{\sigma}\right)$$

To use tables of $\Phi(z)$, take $z = \frac{x-\mu}{\sigma}$.

5.3 Experimental Samples

$x_1, x_2, \ldots x_n$ denote a set of n observations of a random variable having a normal distribution whose population mean μ is unknown.

Range $= x_{max} - x_{min}$

Sample mean $m = \frac{1}{n}\Sigma x_i$

Average deviation $= \frac{1}{n}\Sigma|x_i - m|$

Sample standard deviation = s ,

Sample variance $= s^2 = \frac{1}{n-1}\Sigma(x_i - m)^2$

Distribution of x is $N(\mu,\sigma^2)$

Distribution of m is $N(\mu,\sigma^2/n)$

Distribution of $\frac{m-\mu}{(\sigma/\sqrt{n})}$ is $N(0,1)$

i.e. standard error of sample means $= \frac{\sigma}{\sqrt{n}}$

If population variance σ^2 is known,

95% confidence interval for μ is $m \pm 1.96\ \sigma/\sqrt{n}$

99% " " " " " $m \pm 2.58\ \sigma/\sqrt{n}$

If population variance σ^2 is unknown: $\frac{m-\mu}{s/\sqrt{n}}$ has the t-distribution with n-1 degrees of freedom (t_{n-1}) and the 95% confidence interval for μ is obtained from $m \pm t_c\ s/\sqrt{n}$ and the table.

95% points of the t-distribution

n-1	t_c	n-1	t_c	n-1	t_c
1	12.7	6	2.45	12	2.18
2	4.30	7	2.36	15	2.13
3	3.18	8	2.31	20	2.09
4	2.78	9	2.26	30	2.04
5	2.57	10	2.23	60	2.00
				∞	1.96

Thus for $n > 20$, $m \pm 1.96\ s/\sqrt{n}$ is a good approximation to the population mean with a 95% confidence.

5.4 Combination of Errors

If results are Normally Distributed, the Most Probable Error S_z in the calculated result $z = f(x, y, \text{etc})$, due to the independent standard errors S_x, S_y, etc. in x, y, etc. is given by,

$$(S_z)^2 = \left(\frac{\partial z}{\partial x} S_x\right)^2 + \left(\frac{\partial z}{\partial y} S_y\right)^2 + \ldots \text{ etc.}$$

If the function f consists of multiplied and divided terms ONLY (i.e. no addition or subtraction)

$$\left(\frac{S_z}{z}\right)^2 = \left(n\,\frac{S_x}{x}\right)^2 + \left(m\,\frac{S_y}{y}\right)^2 + \ldots \text{ etc.}$$

where n, m, etc. are the powers of x, y, etc. in f.

Notes

(1) The Maximum Possible Error $\left(\delta z = \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y, \text{ etc.}\right)$ is rarely of interest in engineering

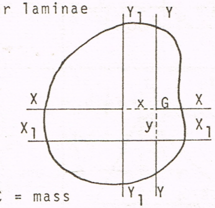
(2) Instrument 'rounding off' error $\pm\delta x$ may be treated as a Normally Distributed error by the equivalence $S_x \doteqdot \frac{2}{3}\delta x$.

6. MECHANICS

Moments of inertia and Second moments of area - General theorems

N.B. The symbol I is used for both second moment of area and moment of inertia.

(i) Parallel axis theorem: Solids or laminae



Centroid is at G

Centre of mass is at G

$$I_{X_1X_1} = I_{XX} + Cy^2$$

$$I_{Y_1Y_1} = I_{YY} + Cx^2$$

where for moment of inertia C = mass

and for second moment of area of a lamina C = area

(ii) Perpendicular axis theorem for laminae:

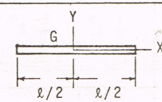
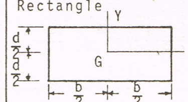
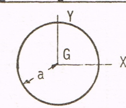
Polar second moment $J_o = I_{XX} + I_{YY} = I_{ZZ}$

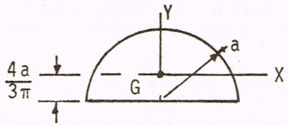
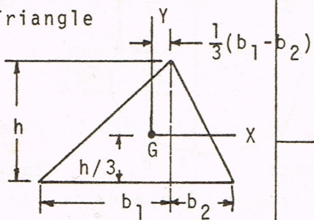
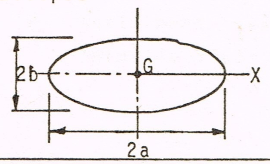
Radii of gyration k

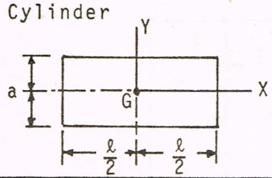
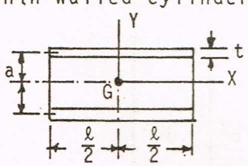
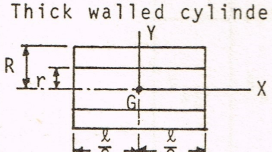
Second moment of area $I = Ak^2$

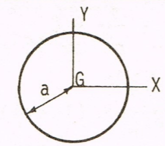
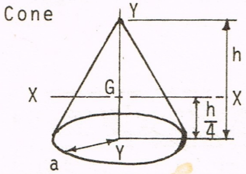
Moment of inertia $I = mk^2$

where A = area, m = mass.

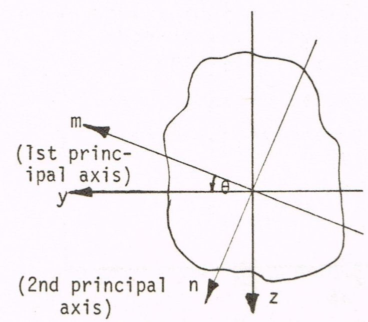
(a) UNIFORM ROD	k^2_{XX}	k^2_{YY}	A
	-	$\frac{\ell^2}{12}$	-
(b) LAMINAE			
Rectangle 	$\frac{1}{12}d^2$	$\frac{1}{12}b^2$	db
Circle 	$\frac{1}{4}a^2$	$\frac{1}{4}a^2$	πa^2

(b) LAMINAE (cont)	k_{XX}^2	k_{YY}^2	A
Semi-circle 	$a^2\left\lvert\frac{1}{4} - \left(\frac{4}{3\pi}\right)^2\right\rvert$	$\frac{1}{4}a^2$	$\frac{\pi a^2}{2}$
Triangle 	$\frac{1}{18}h^2$	$\frac{1}{18}(b_1^2+b_1b_2+b_2^2)$	$\frac{h}{2}(b_1+b_2)$
	$k_{XY}^2 = \frac{1}{36}h(b_1 - b_2)$		
Ellipse 	$\frac{b^2}{4}$	$\frac{a^2}{4}$	πab

(c) SOLIDS	k_{XX}^2	k_{YY}^2 and k_{ZZ}^2	V
Cylinder 	$\frac{1}{2}a^2$	$\frac{1}{4}a^2 + \frac{1}{12}\ell^2$	$\pi a^2\ell$
Thin-walled cylinder 	$a^2 + \frac{1}{4}t^2$	$\frac{1}{2}a^2 + \frac{1}{8}t^2 + \frac{1}{12}\ell^2$	$2\pi at\ell$
Thick walled cylinder 	$\frac{1}{2}(R^2 + r^2)$	$\frac{\ell^2}{12} + \frac{R^2+r^2}{4}$	$\pi(R^2 - r^2)\ell$

(c) SOLIDS (cont)	k_{XX}^2 and k_{ZZ}^2	k_{YY}^2	V
Sphere 	$\frac{2}{5}a^2$	$\frac{2}{5}a^2$	$\frac{4}{3}\pi a^3$
Cone 	$\frac{3(4a^2+h^2)}{80}$	$\frac{3a^2}{10}$	$\frac{\pi}{3}a^2h$

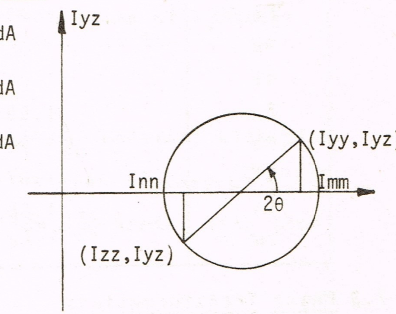
Mohr's Circle for Second Moment of Area



$$Iyy = \int_A z^2\, dA$$

$$Izz = \int_A y^2\, dA$$

$$Iyz = \int_A yz\, dA$$



Constant acceleration equations

$$v = u + at$$

$$v^2 = u^2 + 2ax$$

$$x = ut + \frac{1}{2}at^2$$

Friction

coefficient of static friction $\mu = \tan\phi$

for no slipping $\frac{F}{N} \leqslant \mu$

Accelerations due to rotation

Coriolis $= 2\,\underline{\omega} \times \left(\frac{\partial \underline{r}}{\partial t}\right)$

Central $= \underline{\omega} \times (\underline{\omega} \times \underline{r})$

DRY SLIDING FRICTION COEFFICIENTS

Clutches	0.3-0.4
Brakes (lining)	0.35-0.5
" (pads)	~0.3
Nylon/Steel	0.3-0.5
Filled PTFE/Steel	0.05-0.3
Perspex/Steel	~0.5
Rubber/Steel	0.6-0.9
Rubber/Asphalt	0.5-0.8
Lignum vitae/Steel	~0.1

7. PROPERTIES AND MECHANICS OF SOLIDS

7.1 Bonding

(a) Condon-Morse Equation $V_{total} = \frac{-Ae^2}{r^n} + \frac{B}{r^m} + C$

(b) Ionic Bond Equation $V_o = \frac{-Z_1^* Z_2^* e^2}{4\pi d\varepsilon_o} (1 - \frac{1}{n}) + \Delta E$

(c) Theoretical Density $\rho = \frac{nA}{VN}$

7.2 Atomic sizes in substitutional alloys

Element	Seitz radius r_o (Å) (at 20°C)	Effective valency in solution
Al	1.58	3
Au	1.59	1
Cu	1.41	1
Fe(α)	1.41	?
Mg	1.85	2
Ni	1.38	1
P	1.58	3
Pb	1.95	4
Si	1.67	4
Sn	1.86	4
Zn	1.54	2

7.3 Phase Transformations

Length and volume changes may be related by:-

$(1 + \Delta V/V) = (1 + \Delta L/L)^3$

7.4 Crystallography

(a) In the Miller system:

Specific Plane	(h.k.ℓ)
Family of Planes	{h.k.ℓ}
Specific Direction	[h.k.ℓ]
Family of Directions	<h.k.ℓ>

(b) Inter-planar spacings for Cubics

$$d_{(h.k.\ell)} = \frac{a}{\sqrt{h^2+k^2+\ell^2}} = \frac{a}{\sqrt{N}}$$

(c) Quadratic Forms of Miller Indicies (N values)

Cubic Structure	N values
Simple	1,2,3,4,5,6,8,9,10,11,12,13,14,16,17,18,19,20.....
Face Centred	3,4,8,11,12,16,19,20,24,27,32.....
Body Centred	2,4,6,8,10,12,14,16,18,20,22,24,26,30.....
Diamond	3,8,11,16,19,24,27,32....

7.5 Defects and Diffusion Data

(a) Number of Defects $n = \alpha N e^{\frac{-Q}{kT}}$

(b) Diffusivity $D = D_o e^{\frac{-Q}{kT}}$

Note: these equations may be expressed in terms of R_o rather than k, the value of Q must be quoted in the appropriate units.

(c) Macroscopic Diffusion

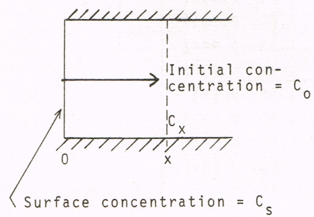
(i) D constant with composition, $\frac{dc}{dx}$ constant with time

$J = - D \frac{dc}{dx}$ (This is a special case of (ii))

(ii) D constant with composition, $\frac{dc}{dx}$ varies with time

$$\frac{dc}{dt} = D \frac{d^2c}{dx^2}$$

Solution for a constant surface potential and impermeable sides



$$\frac{C_x - C_o}{C_s - C_o} = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right)$$

7.6 Selected Values of Error Function $\frac{2}{\sqrt{\pi}}\int_0^z e^{-u^2}du$

z	erfz	z	erfz	z	erfz	z	erfz
0.00	0.0000	0.68	0.6638	1.36	0.9456	2.00	0.9953
0.02	0.0226	0.70	0.6778	1.38	0.9490	2.05	0.9963
0.04	0.0451	0.72	0.6914	1.40	0.9523	2.10	0.9970
0.06	0.0676	0.74	0.7047	1.42	0.9554	2.15	0.9976
0.08	0.0901	0.76	0.7175	1.44	0.9583	2.20	0.9981
0.10	0.1125	0.78	0.7300	1.46	0.9611	2.25	0.9983
0.12	0.1348	0.80	0.7421	1.48	0.9637	2.30	0.9989
0.14	0.1570	0.82	0.7538	1.50	0.9661	2.35	0.9991
0.16	0.1790	0.84	0.7651	1.52	0.9684	2.40	0.9993
0.18	0.2009	0.86	0.7761	1.54	0.9706	2.45	0.9995
0.20	0.2227	0.88	0.7867	1.56	0.9726	2.50	0.9996
0.22	0.2443	0.90	0.7969	1.58	0.9746	2.55	0.9997
0.24	0.2657	0.92	0.8068	1.60	0.9764	2.60	0.9998
0.26	0.2869	0.94	0.8163	1.62	0.9780	2.65	0.9998
0.28	0.3079	0.96	0.8254	1.64	0.9796	2.70	0.9999
0.30	0.3286	0.98	0.8342	1.66	0.9811	2.75	0.9999
0.32	0.3491	1.00	0.8427	1.68	0.9825	2.80	0.9999
0.34	0.3694	1.02	0.8508	1.70	0.9838	2.85	0.9999
0.36	0.3893	1.04	0.8587	1.72	0.9850	2.90	1.0000
0.38	0.4090	1.06	0.8661	1.74	0.9861	2.95	1.0000
0.40	0.4284	1.08	0.8733	1.76	0.9872	3.00	1.0000
0.42	0.4475	1.10	0.8802	1.78	0.9882	4.00	1.0000
0.44	0.4662	1.12	0.8868	1.80	0.9891		
0.46	0.4847	1.14	0.8931	1.82	0.9899		
0.48	0.5028	1.16	0.8991	1.84	0.9907		
0.50	0.5205	1.18	0.9048	1.86	0.9915		
0.52	0.5379	1.20	0.9103	1.88	0.9922		
0.54	0.5549	1.22	0.9155	1.90	0.9928		
0.56	0.5716	1.24	0.9205	1.92	0.9934		
0.58	0.5879	1.26	0.9252	1.94	0.9939		
0.60	0.6039	1.28	0.9297	1.96	0.9944		
0.62	0.6194	1.30	0.9340	1.98	0.9949		
0.64	0.6346	1.32	0.9381				
0.66	0.6494	1.34	0.9419				

7.7 Fracture

i) Fatigue

Manson-Coffin Law $\sqrt{N_p}\,\varepsilon_p = c$ (c = constant)

Miner's Rule $\sum\left(\frac{n_i}{N_i}\right) = 1$

Rayleigh Distribution $P(\sigma) = \sigma r^{-2}\exp\left\{-\frac{1}{2}\left(\frac{\sigma}{r}\right)^2\right\}$

Fraction of peak exceeding stress (σ) expressed in terms of E $E(\sigma) = \exp\left\{-\frac{1}{2}\left(\frac{\sigma}{r}\right)^2\right\}$

ii) Fracture Toughness

Stress Intensity $K = Q\sigma\sqrt{\pi a}$

Paris Equation $\frac{da}{dN} = A(\Delta K)^n = A_1 a^{m/2}$ (A, A_1, n and m are constants)

7.8 Some typical values of physical properties

All values are given, unless otherwise stated, for a temperature of 20°C.

	Carbon Steel	Aluminium Alloys	Brass 65/35	Copper	Concrete	Stainless Steels	Wood
ρ (kg/m^3)	7850	2720	8450	8960	2400	8000	400-800
E (GN/m^2)	207	68.9	105	104	13.8	213	8-13
G (GN/m^2)	79.6	26.5	38.0	46		82	
K (GN/m^2)	172	57.5	115	130		178	
ν	0.3	0.3	0.35	0.35	0.1	0.3	
α (μm/(mK))	11	23	19	11.2		18	~0.15
σ_y (MN/m^2)	230-460	30-280	62-430	47-320		200-585	
σ_f (MN/m^2)	400-770	90-300	330-530	200-350	27-55	500-800	50-100

K for water is 2.3 GN/m^2

The lower values of σ_y and σ_f for carbon and stainless steels refer to materials such as plates and tubes while the higher figures refer to heat-treated material such as used for bolts. The range of values for aluminium, copper and brass is due to the change in material property achieved by heat-treatment and/or mechanical work.

7.6 Selected Values of Error Function $\frac{2}{\sqrt{\pi}}\int_0^z e^{-u^2}du$

z	erfz	z	erfz	z	erfz	z	erfz
0.00	0.0000	0.68	0.6638	1.36	0.9456	2.00	0.9953
0.02	0.0226	0.70	0.6778	1.38	0.9490	2.05	0.9963
0.04	0.0451	0.72	0.6914	1.40	0.9523	2.10	0.9970
0.06	0.0676	0.74	0.7047	1.42	0.9554	2.15	0.9976
0.08	0.0901	0.76	0.7175	1.44	0.9583	2.20	0.9981
0.10	0.1125	0.78	0.7300	1.46	0.9611	2.25	0.9983
0.12	0.1348	0.80	0.7421	1.48	0.9637	2.30	0.9989
0.14	0.1570	0.82	0.7538	1.50	0.9661	2.35	0.9991
0.16	0.1790	0.84	0.7651	1.52	0.9684	2.40	0.9993
0.18	0.2009	0.86	0.7761	1.54	0.9706	2.45	0.9995
0.20	0.2227	0.88	0.7867	1.56	0.9726	2.50	0.9996
0.22	0.2443	0.90	0.7969	1.58	0.9746	2.55	0.9997
0.24	0.2657	0.92	0.8068	1.60	0.9764	2.60	0.9998
0.26	0.2869	0.94	0.8163	1.62	0.9780	2.65	0.9998
0.28	0.3079	0.96	0.8254	1.64	0.9796	2.70	0.9999
0.30	0.3286	0.98	0.8342	1.66	0.9811	2.75	0.9999
0.32	0.3491	1.00	0.8427	1.68	0.9825	2.80	0.9999
0.34	0.3694	1.02	0.8508	1.70	0.9838	2.85	0.9999
0.36	0.3893	1.04	0.8587	1.72	0.9850	2.90	1.0000
0.38	0.4090	1.06	0.8661	1.74	0.9861	2.95	1.0000
0.40	0.4284	1.08	0.8733	1.76	0.9872	3.00	1.0000
0.42	0.4475	1.10	0.8802	1.78	0.9882	4.00	1.0000
0.44	0.4662	1.12	0.8868	1.80	0.9891		
0.46	0.4847	1.14	0.8931	1.82	0.9899		
0.48	0.5028	1.16	0.8991	1.84	0.9907		
0.50	0.5205	1.18	0.9048	1.86	0.9915		
0.52	0.5379	1.20	0.9103	1.88	0.9922		
0.54	0.5549	1.22	0.9155	1.90	0.9928		
0.56	0.5716	1.24	0.9205	1.92	0.9934		
0.58	0.5879	1.26	0.9252	1.94	0.9939		
0.60	0.6039	1.28	0.9297	1.96	0.9944		
0.62	0.6194	1.30	0.9340	1.98	0.9949		
0.64	0.6346	1.32	0.9381				
0.66	0.6494	1.34	0.9419				

7.7 Fracture

i) Fatigue

Manson-Coffin Law $\sqrt{N_p}\,\varepsilon_p = c$ (c = constant)

Miner's Rule $\sum\left(\frac{n_i}{N_i}\right) = 1$

Rayleigh Distribution $P(\sigma) = \sigma r^{-2}\exp\left\{-\frac{1}{2}\left(\frac{\sigma}{r}\right)^2\right\}$

Fraction of peak exceeding stress (σ) expressed in terms of E $E(\sigma) = \exp\left\{-\frac{1}{2}\left(\frac{\sigma}{r}\right)^2\right\}$

ii) Fracture Toughness

Stress Intensity $K = Q\sigma\sqrt{\pi a}$

Paris Equation $\frac{da}{dN} = A(\Delta K)^n = A_1 a^{m/2}$ (A, A_1, n and m are constants)

7.8 Some typical values of physical properties

All values are given, unless otherwise stated, for a temperature of 20°C.

	Carbon Steel	Aluminium Alloys	Brass 65/35	Copper	Concrete	Stainless Steels	Wood
ρ (kg/m^3)	7850	2720	8450	8960	2400	8000	400-800
E (GN/m^2)	207	68.9	105	104	13.8	213	8-13
G (GN/m^2)	79.6	26.5	38.0	46		82	
K (GN/m^2)	172	57.5	115	130		178	
ν	0.3	0.3	0.35	0.35	0.1	0.3	
α (μm/(mK))	11	23	19	11.2		18	~0.15
σ_y (MN/m^2)	230-460	30-280	62-430	47-320		200-585	
σ_f (MN/m^2)	400-770	90-300	330-530	200-350	27-55	500-800	50-100

K for water is 2.3 GN/m^2

The lower values of σ_y and σ_f for carbon and stainless steels refer to materials such as plates and tubes while the higher figures refer to heat-treated material such as used for bolts. The range of values for aluminium, copper and brass is due to the change in material property achieved by heat-treatment and/or mechanical work.

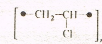
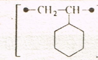
Metals

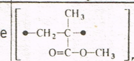
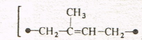
Property	Copper	Iron
Crystal structure	f.c.c.	b.c.c.
Bonding	metallic	metallic
Lattice constant (Å)	3.61	2.86
Atomic volume (m^3/kg mol)	7.09×10^{-3}	7.10×10^{-3}
ρ (kg/m^3)	8.96×10^3	7.87×10^3
Resistivity (Ω m)	1.72×10^{-8}	10×10^{-8}
Cohesive energy (J/kg mol)	3.38×10^8	4.05×10^8
Melting point (^{o}C)	1083	1530
α(μm/(mK))	16.7	12.1
Fermi energy (eV)	7.04	11.2
Work function (eV)	4.07 - 4.18	3.91 - 4.77
Temperature coefficient of resistance (K^{-1})	+0.0043	+0.0065
Effective radius (Å) of		
(a) neutral atom	1.27	1.26
(b) singly charged ion	0.96	-
(c) doubly charged ion	0.70	0.75

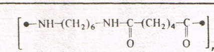
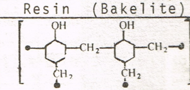
Semiconductors

Property	Germanium	Silicon
Crystal structure	diamond	diamond
Bonding	covalent	covalent
Lattice constant (Å)	5.6575	5.4307
Atomic volume (m^3/kg mol)	13.5×10^{-3}	12.0×10^{-3}
Density (kg/m^3)	5.32×10^3	2.33×10^3
Cohesive energy (J/kg mol)	3.72×10^8	4.39×10^8
Melting point (^{o}C)	958.5	1412
Mobility (m^2/(V s))	electrons 0.38 holes 0.18	electrons 0.19 holes 0.05
Energy gap (eV) (room temperature)	0.67	1.107
Density of states effective mass	electrons 0.35 m_e holes 0.56 m_e	electrons 0.58 m_e holes 1.06 m_e
α(μm/(mK))	5.75	7.6

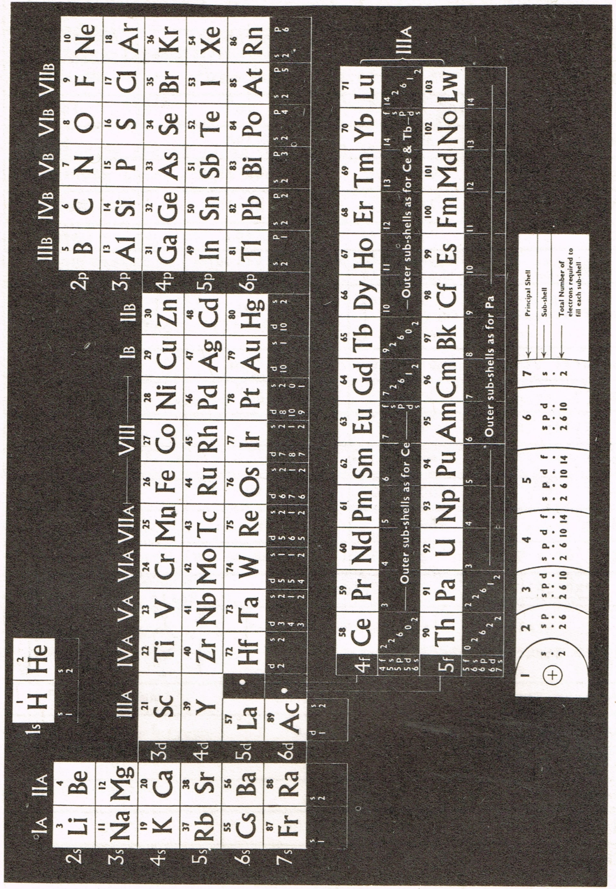
Polymers

PROPERTY	Polyethylene (H.D)	Polyvinyl Chloride	Polystyrene
Polymer Structure	$[\bullet\!-CH_2-CH_2-\!\bullet]_n$		
Structural State	Crystalline	Amorphous/ Slightly Crystalline	Amorphous/ Crystalline
ρ(kg/m^3)	0.96×10^3	1.7×10^3	1.05×10^3
Resistivity (Ωm)	$10^6 - 10^{10}$	10^5	10^{10}
α(μm/mK)	120	190	63
E(GN/m^2) M?	70-280	2500-3500	3500-4200
σ_f(MN/m^2)	7-14	28-40	35-50
Tg(K)	153	353	373

PROPERTY	Polymethyl-methacrylate	Polytetrafluor-ethylene	Polyisoprene (Natural Rubber)
Polymer Structure		$[\bullet\!-CF_2-CF_2-\!\bullet]_n$	
Structural State	Amorphous	Crystalline	Elastomer
ρ(kg/m^3)	1.2×10^3	2.2×10^3	1.5×10^3
Resistivity (Ωm)	10^8	10^8	$10^5 - 10^7$
α(μm/mK)	90	100	-
E(GN/m^2) M?	2500-4000	400-650	7-70
σ_f(MN/m^2)	50-70	14-30	2-10
Tg(K)	380	399	203

PROPERTY	Nylon 6:6	Phenol-Formaldehyde Resin (Bakelite)
Polymer Structure		
Structure State	Crystalline	Amorphous
ρ(kg/m^3)	1.15×10^3	1.3×10^3
Resistivity (Ωm)	10^6	10^4
α(μm/mK)	100	72
E(GN/m^2)	2000-3000	7000
σ_f(MN/m^2)	50-70	50
Tg(K)	323	-

Periodic Table of the Elements



1. The atomic number indicates the number of protons in the nucleus of an atom. In the neutral atom these protons are electrically balanced by an equal number of electrons outside the nucleus. Only neutral atoms are considered in the Periodic Classification.
2. Electrons travel far from the nucleus but if those regions where they spend most of their time are considered, a well-defined pattern of layers or 'Principal Shells' appears. Each shell is known by a Principal Quantum Number 1, 2, 3 . . . 7 or sometimes by the letters K, L, M . . . etc.
3. In each shell the electrons move around the nucleus in complicated, three-dimensional patterns called Orbitals. The laws of Quantum Mechanics permit only certain types of orbital. An electron following one of these paths possesses an amount of energy (Energy Level) characteristic of that orbital.
4. Four types of orbital are encountered; they are identified by the letters *s p d* and *f*. *s* is the simplest whilst *p*, *d* and *f* are progressively more complex.
5. The number of orbitals per shell increases with shell number. (See the lower diagram overleaf.) The first contains only an *s* orbital, the second an *s* and three *p*'s, the third adds five *d* orbitals and the fourth seven *f*'s. These groups of like orbitals in any Principal Shell are called *s p d* or *f* sub-shells.

   Each sub-shell, depending on its principal quantum number and type, has a characteristic energy the order of which is generally proportional to the distance of the sub-shell from the nucleus.
6. Each orbital accepts either one or two electrons and the maximum number of electrons per sub-shell is shown on the diagram.
7. Electrons take positions in orbitals where the energy level is lowest. Up to element 18 (Argon) sub-shells and shells are built in an orderly sequence to maximum capacity. But in the next group the order changes because it happens that the energy level of the 4 *s* state is a little lower than that of the 3 *d* state.
8. The first transition series begins with Scandium (element 21) where the energy levels of the 4 *s* and 3 *d* orbitals are so nearly equal that there is a tendency for electrons to move from one orbital to another, causing variable valency. The same happens in the fifth period with 5 *s* and 4 *d* orbitals and in the sixth period with the 6 *s* and 5 *d* orbitals.
9. In the Lanthanide and Actinide series of elements, the 4 *f* and 5 *f* orbitals are occupied only after the *s p d* and *s* orbitals outside them have filled or begun to fill. The effect upon the chemistry of the elements is very small because the *f* orbitals are deep in the core of the atom. For this reason there is little difference between one element and its immediate neighbours.
10. In any element, the so called Valency Electrons are those moving in orbitals of the highest energy levels. In this Chart of the Periodic Classification, the number and position of the valency electrons is indicated in the boxes underneath the various columns e.g. Rhodium—element 45—has nine valency electrons; 8 in the 4 *d* sub-shell and 1 in the 5 *s*.

    The particular sub-shell being filled with electrons is shown by the figures 4 *s*, 3 *d*, 4 *p* etc. in front of the rows of elements e.g. the 3 *d* in front of elements 21—30.

NOTES

1. The following atomic weights are based on the exact number 12 for the carbon isotope 12, as agreed between the International Unions of Pure and Applied Physics and of Pure and Applied Chemistry, 1961.

2. The values given normally indicate the mean atomic weight of the mixture of isotopes found in nature. Particular attention is drawn to the value for hydrogen, boron, carbon, oxygen, silicon and sulphur, where the deviation shown is due to variation in relative concentration of isotopes.

Symbol	Name	Atomic Number	Atomic Weight
A or Ar	Argon	18	39·948
Ac	Actinium	89	—
Ag	Silver	47	107·870
Al	Aluminium	13	26·9815
Am	Americium	95	—
As	Arsenic	33	74·9216
At	Astatine	85	—
Au	Gold	79	196·967
B	Boron	5	10·811 ±0·003
Ba	Barium	56	137·34
Be	Beryllium	4	9·0122
Bi	Bismuth	83	208·980
Bk	Berkelium	97	—
Br	Bromine	35	79·909
C	Carbon	6	12·01115 ±0·00005
Ca	Calcium	20	40·08
Cd	Cadmium	48	112·40
Ce	Cerium	58	140·12
Cf	Californium	98	—
Cl	Chlorine	17	35·453
Cm	Curium	96	—
Co	Cobalt	27	58·9332
Cr	Chromium	24	51·996
Cs	Caesium	55	132·905
Cu	Copper	29	63·54
Dy	Dysprosium	66	162·50
Er	Erbium	68	167·26
Es	Einsteinium	99	—
Eu	Europium	63	151·96
F	Fluorine	9	18·9984
Fe	Iron	26	55·847
Fm	Fermium	100	—
Fr	Francium	87	—
Ga	Gallium	31	69·72
Gd	Gadolinium	64	157·25
Ge	Germanium	32	72·59
H	Hydrogen	1	1·00797 ±0·00001
He	Helium	2	4·0026
Hf	Hafnium	72	178·49
Hg	Mercury	80	200·59
Ho	Holmium	67	164·930
	Iodine	53	126·9044
In	Indium	49	114·82
Ir	Iridium	77	192·2
K	Potassium	19	39·102
Kr	Krypton	36	83·80
La	Lanthanum	57	138·91
Li	Lithium	3	6·939
Lu	Lutetium	71	174·97
Md	Mendeleevium	101	—
Mg	Magnesium	12	24·312
Mn	Manganese	25	54·9380
Mo	Molybdenum	42	95·94
N	Nitrogen	7	14·0067
Na	Sodium	11	22·9898
Nb	Niobium	41	92·906
Nd	Neodymium	60	144·24
Ne	Neon	10	20·183
Ni	Nickel	28	58·71
No	Nobelium	102	—
Np	Neptunium	93	—
O	Oxygen	8	15·9994 ±0·0001
Os	Osmium	76	190·2
P	Phosphorus	15	30·9738
Pa	Protoactinium	91	—
Pb	Lead	82	207·19
Pd	Palladium	46	106·4
Pm	Promethium	61	—
Po	Polonium	84	—
Pr	Praseodymium	59	140·907
Pt	Platinum	78	195·09
Pu	Plutonium	94	—
Ra	Radium	88	—
Rb	Rubidium	37	85·47
Re	Rhenium	75	186·2
Rh	Rhodium	45	102·905
Rn	Radon	86	—
Ru	Ruthenium	44	101·07
S	Sulphur	16	32·064 ±0·003
Sb	Antimony	51	121·75
Sc	Scandium	21	44·956
Se	Selenium	34	78·96
Si	Silicon	14	28·086 ±0·001
Sm	Samarium	62	150·35
Sn	Tin	50	118·69
Sr	Strontium	38	87·62
Ta	Tantalum	73	180·948
Tb	Terbium	65	158·924
Tc	Technetium	43	—
Te	Tellurium	52	127·60
Th	Thorium	90	232·038
Ti	Titanium	22	47·90
Tl	Thallium	81	204·37
Tm	Thulium	69	168·934
U	Uranium	92	238·03
V	Vanadium	23	50·942
W	Tungsten	74	183·85
Xe	Xenon	54	131·30
Y	Yttrium	39	88·905
Yb	Ytterbium	70	173·04
Zn	Zinc	30	65·37
Zr	Zirconium	40	91·22

8. THERMODYNAMICS AND FLUID MECHANICS

8.1 Thermodynamic Relationships

1st Law: $dQ - dW = dU$

Enthalpy: $H = U + pV$ or $h = u + pv$

For reversible process: $dS = \left(\frac{dQ}{T}\right)_{rev}$ or $dQ = TdS$

" " ": $dW = pdV$

Helmholtz function: $F = U - TS$ or $f = u - Ts$

Gibbs function / Gibbs free energy: $G = H - TS$ or $g = h - Ts$

From 1st Law for a homogeneous fluid: $Tds = du + pdv = dh - vdp$

Specific heat at constant volume: $c_v = \left(\frac{\partial u}{\partial T}\right)_v$

Specific heat at constant pressure: $c_p = \left(\frac{\partial h}{\partial T}\right)_p$

Specific heat ratio: $\gamma = c_p/c_v$

Reversible engine (Carnot) efficiency: $= 1 - (T_{sink}/T_{source})$

Engine indicated Power: $P_i = p_m V_s N_c$

Steady flow energy equation: $(Q-W)/m = h_2 - h_1 + \frac{1}{2}(c_2^2 - c_1^2) + g(z_2 - z_1)$

Continuity equation: $\dot{m} = \rho A c$

General relationships for a perfect gas:

$pv = RT$ $\qquad pV = mRT$

$pv_o = R_oT$ $\qquad MR = R_o$

$\Delta U = mc_v(T_2 - T_1)$ $\qquad \Delta H = mc_p(T_2 - T_1)$

$$\Delta S = mc_v \ln\left(\frac{p_2}{p_1}\right) + mc_p \ln\left(\frac{v_2}{v_1}\right)$$

$c_p - c_v = R$ $\qquad \frac{\gamma - 1}{\gamma} = \frac{R}{c_p}$

Van der Waals' equation $(p + \frac{a}{v^2})(v - b) = RT$

$S = k \ln P$ $\qquad k = R_o/N$

Availability, (closed system): $(A_1 - A_o) = (U_1 + p_oV_1 - T_oS_1) - (U_o + p_oV_o - T_oS_o)$

" (flow process) : $(B_1 - B_o) = (H_1 - T_oS_1) - (H_o - T_oS_o)$

Maximum work of a Reaction $W_{max} = G_{react} - G_{prod} = R_oT \ln(K_p . p^n)$

For reversible polytropic (pV^n = constant) closed system:

$$W = (p_1V_1 - p_2V_2)/(n-1)$$

For perfect gas also:

$$W = mR(T_1 - T_2)/(n-1) \qquad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$Q = \frac{\gamma - n}{\gamma - 1}.W$$

For adiabatic reversible (isentropic reversible):

$$n = \gamma$$

For isothermal reversible:

$$W = Q = pV\ \ell n\left(\frac{V_1}{V_2}\right) \qquad (n = 1)$$

Maxwell relations

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v \qquad \left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \qquad \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$$

Heat transfer

Conduction (one dimensional) $\dot{Q}/A = -k\,dT/dx$

$$= k\left(T_1 - T_2\right)/x_{1,2}$$

" (radial flow) $\dot{Q}/\ell = 2\pi k\Delta T/\ell n(r_2/r_1)$

Forced convection in a tube Nu $= .023\ Re^{0.8}Pr^{0.4}$
(characteristic length = hydraulic mean diameter)(see 8.5)

Log. mean temperature difference $\dfrac{\Delta T_{in} - \Delta T_{out}}{\ell n(\Delta T_{in}/\Delta T_{out})} = \Delta T_m$

Stefan-Boltzmann Law of radiation $q_b = \sigma T^4$

Radiation exchange:

Grey body to black or large enclosure $\dot{Q}/A = \sigma\varepsilon_1\left(T_1^4 - T_2^4\right)$

Large parallel grey surfaces $\dot{Q}/A = \dfrac{\sigma\left(T_1^4 - T_2^4\right)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$

Heat transfer coefficient $h = \dot{Q}/A\Delta T$

emissivity $\varepsilon = q/q_b$

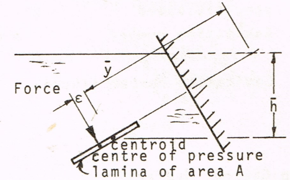
Fluid Mechanics

Statics

$$\frac{\partial p}{\partial z} = -g\rho$$

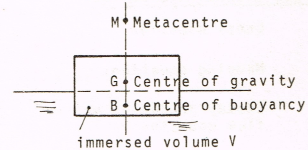
Force $= g\rho A\bar{h}$

$$\varepsilon = \frac{(Ak^2)\text{centroid}}{A\bar{y}}$$



$$\overline{GM}_{roll} = \frac{Ak^2}{V} - \overline{BG}$$

(Ak^2 is 2nd moment of area about rolling axis)



Dynamics

For simple Newtonian flow $\tau = \mu\dfrac{dV}{dy}$

Euler's equation $\dfrac{1}{\rho}\dfrac{dp}{dx} + c\dfrac{dc}{dx} + g\dfrac{dz}{dx} = 0$

Bernoulli's equation $\dfrac{p}{\rho g} + \dfrac{c^2}{2g} + z = \text{constant}$

For constant area flow with friction (Fanno)

$$\frac{dp}{\rho} + c\ dc + 2\frac{fc^2}{D}\,d\ell = 0$$

Acceleration along a stream-line $a_s = V_s\dfrac{\partial V_s}{\partial s} + \dfrac{\partial V_s}{\partial t}$

Acceleration normal to a streamline $a_n = \dfrac{V_s^2}{r} + \dfrac{\partial V_n}{\partial t}$

Reynolds' Equation for bearings:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right) = \frac{1}{2}\frac{\partial}{\partial x}(\rho Uh) + \frac{\partial}{\partial t}(\rho h) + \rho W$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\rho r h^3}{12\eta}\frac{\partial p}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial \phi}\right) = \frac{1}{2}\frac{\partial}{\partial \phi}(\rho h\omega) + \frac{\partial}{\partial t}(\rho h) + \rho W$$

Hydraulic machines

Head coefficient $\psi = Y/\omega^2D^2$

Flow " $\phi = Q/\omega D^3$

Dimensionless specific speed $n_s = \left|\ \omega Q^{\frac{1}{2}}/Y^{\frac{3}{4}} = \phi^{\frac{1}{2}}/\psi^{\frac{3}{4}} = \dfrac{\omega(\text{Power})^{\frac{1}{2}}}{\rho^{\frac{1}{2}}\,Y^{5/4}}\right|_{\eta_{MAX}}$

Dimensionless diameter $\Delta = DY^{\frac{1}{4}}/Q^{\frac{1}{2}}$

Dimensionless suction specific speed $N_{ss} = \omega Q^{\frac{1}{2}}/(NPSE)^{\frac{3}{4}}$

Cavitation number σ or $k = (P_\infty - P_v)/(\frac{1}{2}\rho V_\infty^2)$, suffix ∞, reference condition.

Cavitation number (Thoma) $\sigma_{Th} = (P_1 - P_v)/(P_2 - P_1)$

suffix 1, abs. pressure at lp side of machine; suffix 2, abs pressure at hp side of machine; suffix v, vapour pressure

Open Channel Hydraulics

Chézy equation $V = C\sqrt{RS}$

Manning equation: $V = \frac{1}{n} R^{2/3} S^{1/2}$

Steady gradually varied flow equation: $\frac{dd}{dx} = \dfrac{S_o - S_f}{1 - \alpha\frac{V^2}{2g}}$ (rectangular channel)

Unsteady gradually varied flow equation: $\frac{\partial d}{\partial x} + \frac{V}{g}\frac{\partial V}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} = S_o - S_f$

Continuity equation: $A\frac{\partial V}{\partial x} + V\frac{\partial A}{\partial x} + T\frac{\partial d}{\partial t} = 0$

(no local inflow or outflow)

Conjugate depths in hydraulic jump: $\frac{d_2}{d_1} = \frac{1}{2}\left(\sqrt{1+8F_1^2} - 1\right)$ (rectangular channel)

High speed gas flow

Nozzles:

Mass flow given by $\dot{m} = AC_d\sqrt{\frac{2n}{(n-1)}\, p_o\, \rho_o\left[\left(\frac{p}{p_o}\right)^{2/n} - \left(\frac{p}{p_o}\right)^{\frac{n+1}{n}}\right]}$

Critical pressure ratio $\frac{p^*}{p_o} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$

Sonic velocity $a = \sqrt{np/\rho}$

where $n \simeq 1.3$ for steam, initially superheated

$\simeq 1.135$ for steam, initially wet or dry saturated

$= \gamma \simeq 1.4$ for air

For perfect gas:

Stagnation temperature $T_o = T\left[1 \mp \frac{(\gamma-1)}{2} M^2\right]$

For air in isentropic flow ($\gamma = 1.4$) $\frac{\dot{m}\sqrt{T_o}}{A^* p_o} = 0.0404\ \frac{kgK^{\frac{1}{2}}}{Ns}$

Turbine Ellipse Law $\frac{\dot{m}\sqrt{T_o}}{Ap_o} = \left[1 - \left(\frac{p}{p_o}\right)^2\right]^{\frac{1}{2}}$

8.5 Dimensionless groups

Drag coefficient $C_D \equiv$ drag force$/\frac{1}{2}\rho V^2 A$

Discharge coefficient

$$C_d \equiv Q_{actual}\Big/\left\{A_{throat}\left\{\frac{2\Delta p_{meter}/\rho}{1 - \left(\frac{A_{throat}}{A_{pipe}}\right)^2}\right\}^{\frac{1}{2}}\right\}$$

Fourier number $Fo \equiv (k/\rho c_p)t/L^2$

Froude number $Fr \equiv V/\sqrt{Lg}$

Grashof number $Gr \equiv g\beta\Delta T L^3\rho^2/\mu^2$

Mach number $M \equiv V/a$

Nusselt number $Nu \equiv hL/k$

Prandtl number $Pr \equiv \mu c_p/k$

Reynolds' number - general $Re \equiv \rho VL/\mu$

" " - rotating disc $Re \equiv \rho\,\omega\, D^2/4\mu$

Weber number $We \equiv V(\rho\ell/\sigma)^{\frac{1}{2}}$

Pipeflow friction factor $f \equiv gDh_f/2\ell V^2$ (round pipes)

$2gmh_f/\ell V^2$ (non-circular duct)

Wall shear stress coefficient $f \equiv \tau_w/\frac{1}{2}\rho V^2$

8.6 Composition of air

	Vol. Analysis	Grav. Analysis
Nitrogen (N_2 - 28.013)	0.7809	0.7553
Oxygen (O_2 - 31.999)	0.2095	0.2314
Argon (A_r - 39.948)	0.0093	0.0128
Carbon dioxide (CO_2 - 44.010)	0.0003	0.0005

Mean Molecular Weight M = 28.96

Specific Gas Constant R = 0.2871 kJ/(kgK)

8.7 Temperatures at the primary fixed points

Normal boiling point of oxygen (oxygen point)	$-182.97^\circ C$
Triple point of water	$0.01^\circ C$
Normal boiling point of water (steam point)	$100.00^\circ C$
Normal boiling point of sulphur (sulphur point)	$444.6^\circ C$
Normal melting point of silver (silver point)	$960.8^\circ C$
Normal melting point of gold (gold point)	$1063^\circ C$

8.8 Critical constants

	molecular weight	T_c (K)	P_c (bar) ($10^5 N/m^2$)	ρ_c (kg/m^3)
hydrogen	2.02	33.3	13.0	31
helium (4)	4.00	5.3	2.29	69.3
water vapour	18.02	647.30	221.2	318.3
nitrogen	28.01	126.1	33.9	311
oxygen	32.00	154.4	50.4	430
carbon dioxide	44.01	304.15	73.8	468

8.9 Approximate physical properties at 20°C, 1 bar ($10^5 N/m^2$)

	R $\frac{kJ}{kgK}$	ρ $\frac{kg}{m^3}$	c_p $\frac{kJ}{kgK}$	c_p/c_v	μ $\frac{mNs}{m^2}$ = cP	k $\frac{W}{mK}$
hydrogen	4.16	0.082	14.3	1.40	8.8×10^{-3}	1.8×10^{-1}
helium	2.08	0.164	5.23	1.66	1.96×10^{-2}	1.4×10^{-1}
nitrogen	0.294	1.16	1.04	1.40	1.76×10^{-2}	2.6×10^{-2}
oxygen	0.260	1.31	0.91	1.40	2.03×10^{-2}	2.6×10^{-2}
carbon dioxide	0.190	1.80	0.84	1.28	1.47×10^{-2}	1.7×10^{-2}
air	0.287	1.19	1.005	1.40	1.82×10^{-2}	2.6×10^{-2}

(ii) Liquids

	ρ kg/m^3	c_p kJ/(kgK)	μ cP	k W/(mK)	σ N/m	β $10^{-3}K^{-1}$
water	1,000	4.19	1.002	0.6	0.073	0.21
mercury	13,600	0.14	1.55	8.7	0.51	0.18
castor oil	960	2.20	1000	0.18	0.039	
benzene	880	1.80	0.656	0.16	0.029	
ethyl alcohol	790	2.86	1.20	0.19	0.022	1.08
engine oil∼	890	1.9	80	0.15	-	0.8
Freon 12	1,350	0.96	0.273	0.073	-	

(iii) Solids

	ρ kg/m^3	c_p kJ/(kgK)	k W/(mK)	α μm/(mK)
duralumin	2720	0.88	170	23
mild steel	7850	0.46	52	11
stainless steel (18% Ni, 8% Cr)	7810	0.46	16	18
brass (65/35)	8450	0.37	120	19
concrete	2400	0.88	1.1	10-14
wood (pine)	500	2.8	0.15	0.15
firebrick	170	0.81	0.38	3-9

(iv) Fuels

(a) Gases (fuels)

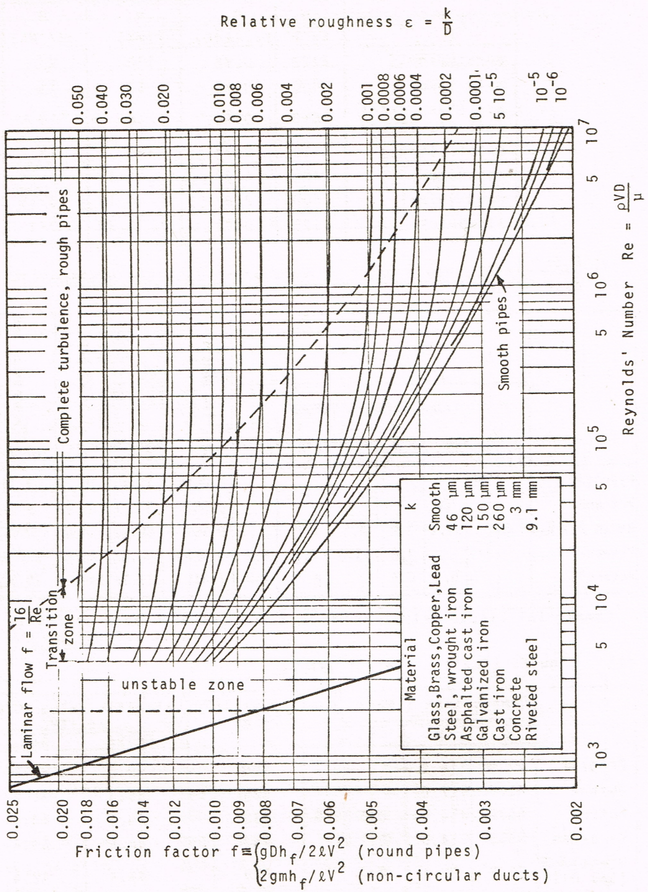
	Composition by Volume							Relative Density (Air) = 1	Calorific Value MJ/m^3 15°C 1.01325 bar		Theoretical Air
	N_2	H_2	CH_4	C_2H_6	C_3H_8	C_4H_{10}	C_3H_6		Gross	Net	Vol/Vol
Hydrogen		100						0.0696	12.10	10.22	2.38
Methane			100					0.5537	37.71	33.95	9.52
North Sea Gas	1.5		94.4	3.0	0.5	0.2		0.589	38.62	34.82	9.75
Propane*				1.5	91.0	2.5	5.0	1.523	93.87	86.43	23.76
Butane*			0.1	0.5	7.2	87.0	4.2	1.941	117.75	108.69	29.92

*Commercial Liquid petroleum Gas (L.P.G.) See also data on liquid fuels below.

(b) Liquids (fuels), typical values

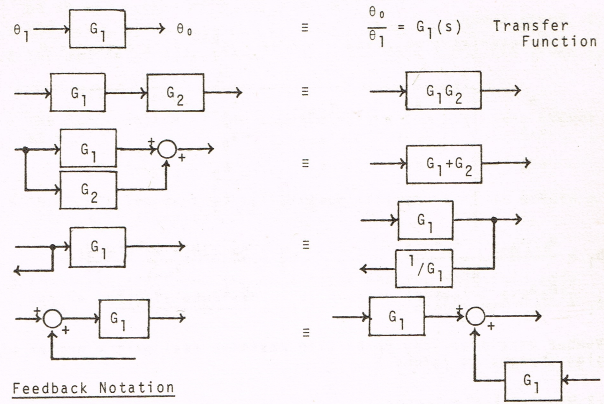
	Composition % Mass			Density at 15°C kg/m^3	Calorific Value MJ/kg at 15°C	
	C	H	S		Gross	Net
Propane*	82.0	18.0		505	50.0	46.3
Butane*	81.9	17.0		575	49.3	45.8
Petrol	85.5	14.4	0.1	733	46.9	43.7
Kerosene	85.9	14.0	0.1	780	46.5	43.4
Diesel (Gas Oil)	85.7	13.4	0.9	840	45.4	42.4

8.10 Friction factor

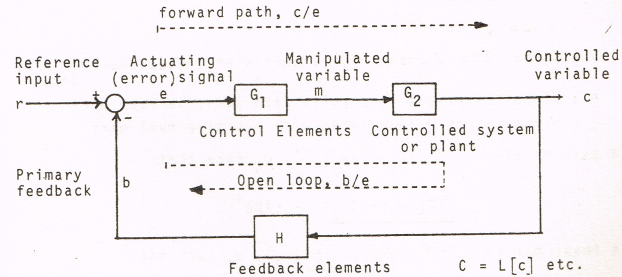


9. AUTOMATIC CONTROL

1. Block Diagrams



2. Feedback Notation



$C = L[c]$ etc.

Input/Output closed loop transfer function $= \frac{C}{R} = \frac{G_1 G_2}{1+G_1 G_2 H}$

Input/Error closed loop transfer function $= \frac{E}{R} = \frac{1}{1+G_1 G_2 H}$

Characteristic equation $= 1+G_1 G_2 H = 0$

$$G_1 G_2 H \triangleq KGH = \frac{K}{s^{\ell}} \frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{i=1}^{n-\ell}(s+p_i)}$$

$-z_i, -p_i$	zeros and poles
ℓ	system type number
n	system order number

3. Stability Criteria for Linear Systems

3.1 Root location: No closed loop system pole may have positive real part

3.2 Routh Array

Characteristic equation $a_n s^n + a_{n-1} s^{n-1} \ldots + a_1 s + a_o = 0$

	1	2	3	
n	a_n	a_{n-2}	a_{n-4}	"
$n-1$	a_{n-1}	a_{n-3}	a_{n-5}	"
$n-2$	b_1	b_2	b_3	"
$n-3$	c_1	c_2	c_3	"
"	"	"	"	"

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

$$c_1 = \frac{b_1 a_{n-3} - a_{n-1}b_2}{b_1} \qquad c_2 = \frac{b_1 a_{n-5} - a_{n-1}b_3}{b_1} \qquad \text{etc.}$$

Number of closed loop poles with positive real part = number of sign changes in column 1.

3.3 Nyquist Encirclement

$$P = N + Z$$

N = number of clockwise encirclements of $(-1,j0)$ by open loop locus

P = number of closed loop poles with positive real part

Z = number of open loop poles with positive real part

3.4 Gain Margin $= |KG(j\omega_g)H(j\omega_g)|^{-1}$, ω_g such that

$$\underline{/KG(j\omega_g)H(j\omega_g)} = -180^0$$

3.5 Phase Margin $= 180^0 + \underline{/KG(j\omega_p)H(j\omega_p)}$, ω_p such that

$$|KG(j\omega_p)H(j\omega_p)| = 1$$

4. Rules of Root Locus Sketching

4.1 Every point, α, on the root locus for positive K satisfies

$$|G(\alpha)H(\alpha)| = 1/K$$

$$\underline{/G(\alpha)H(\alpha)} = (1+2k)180^0 \qquad k = 0, \pm1, \pm2.....$$

4.2 The number of branches of the root locus is equal to the number of poles.

4.3 Branches of the locus can be considered to start on the poles ($K = 0$) and terminate on zeros ($K = \infty$).

4.4 Points of the root locus exist on the real axis to the left of an odd number of poles plus zeros.

4.5 The locus is symmetrical with respect to the real axis.

4.6 The angles of asymptotes, α_k, to the root locus are given by

$$\alpha_k = \frac{\pm(2k+1)\pi}{n-m} \qquad k = 0, 1, 2.....$$

4.7 The intersection of the asymptotes and the real axis occurs at s_r

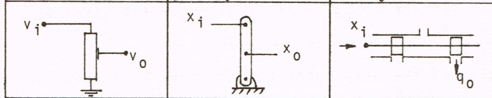
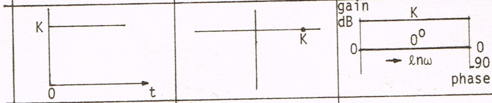
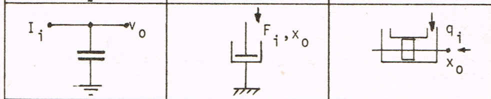
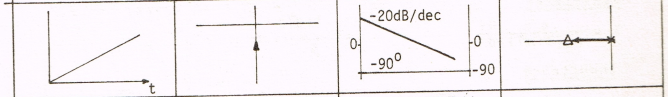
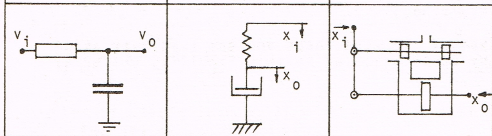
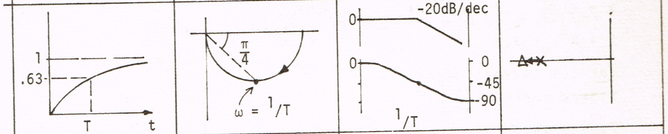
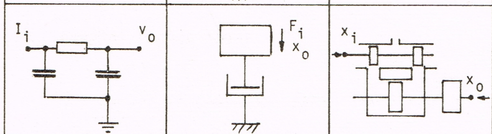
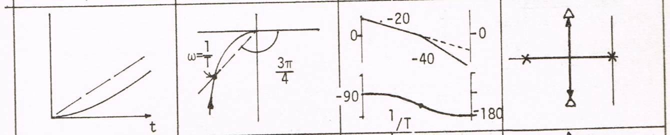
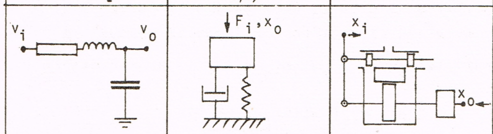
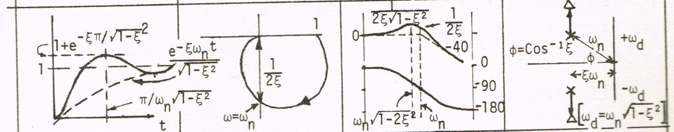
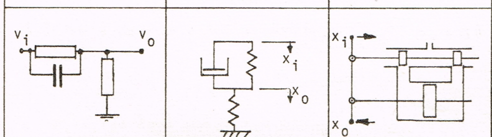
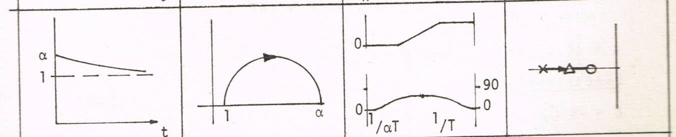
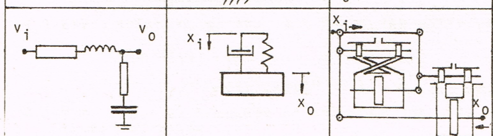
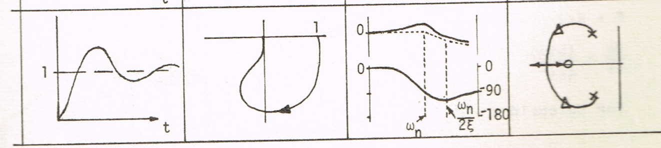
$$s_r = -\frac{\Sigma p_i - \Sigma z_i}{n-m}$$

4.8 The locus leaves the real axis or arrives at it at points α where α is given by

$$\frac{d}{d\alpha}[\ell n\{KG(\alpha)H(\alpha)\}] = 0$$

4.9 The intersection of the root locus and the imaginary axis can be found by application of Routh's Stability criteria.

5. TABLE OF CHARACTERISTIC SYSTEMS

Typical Example (Electrical, Dynamic, Hydraulic)	Basic form of transfer function G(s)	Step Response, Frequency Response (Complex Plane (Nyquist), Logarithmic (Bode)), Pole-Zero Map and Root Locus
	K	 Not Applicable
	$\frac{1}{s}$	
	$\frac{1}{1+Ts}$	
	$\frac{1}{s(1+Ts)}$	
	$\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$	
	$\frac{1+\alpha Ts}{1+Ts}$	
	$\frac{\omega_n^2(1+2\xi s/\omega_n)}{s^2+2\xi\omega_n s+\omega_n^2}$	

10. ELECTRICITY

Ohm's Law

$V = IR, \quad R = \frac{V}{I}, \quad I = \frac{V}{R}$

Power

DC Power $= VI = I^2R = V^2/R$

AC Power $= \mathrm{Re}(\underline{V}.\underline{I}) = |V||I|\cos\phi$

Resistance

$I = \frac{a}{\rho_o(1+\alpha T)}\frac{dV}{dx} \qquad R = \int \frac{\rho_o(1+\alpha T)}{a}\,dx$

Inductance

$e = -L\frac{di}{dt} \qquad i = -\int \frac{V}{L}\,dt$

$L = N^2\mu_o\mu_r a/\ell$

for L R circuit decay $i = Ie^{-Rt/L}$

Stored energy $= \frac{1}{2}LI^2$

Capacitance

$Q = CV = \int idt$

$i = \frac{dQ}{dt} = C\frac{dV}{dt}$

$C = \varepsilon_o\varepsilon_r(n-1)a/d$, for n parallel plates

$\varepsilon_o = 8.85.10^{-12}\ \mathrm{Fm}^{-1}$

for RC circuit discharge $i = -Ie^{-t/RC}$

Stored energy $= \frac{1}{2}CV^2$

$F = \frac{1}{2}\varepsilon_o\varepsilon_r a\left(\frac{V}{x}\right)^2$

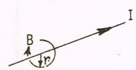
Electrostatics

$F = \frac{Q_1Q_2}{4\pi\varepsilon_o r^2} \qquad \underline{F} = e\underline{E} = -e\,\mathrm{grad}V$

$Q = \oint \underline{D}.d\underline{S}\,(=\Psi) \qquad \underline{D} = \varepsilon_o\varepsilon_r\underline{E}$

Electromagnetism

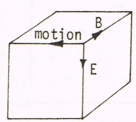
$E = -N\frac{d\Phi}{dt} \qquad B = \frac{\mu_o I}{2\pi r}$



$F = B\ell I \qquad F = \frac{\mu_o I_1 I_2 \ell}{2\pi d}$

$\frac{dH}{d\ell} = \frac{I\sin\alpha}{4\pi x^2}$

For solenoid $H = \frac{NI}{\ell}$



Magnetism

$H = \frac{B}{\mu_o\mu_r}$

For a magnetic circuit

$B = \frac{\Phi}{a}$

$\Phi = \frac{NI}{\frac{\ell_1}{\mu_1 a_1} + \frac{\ell_2}{\mu_2 a_2}}$

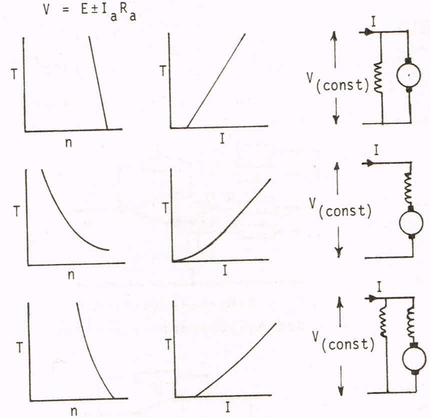
Stored Energy Density $= \frac{1}{2}HB = \frac{1}{2}\frac{B^2}{\mu_o}$

$F = (\frac{1}{2}HB)a = \frac{B^2 a}{2\mu_o}$

DC Machines

$E = \frac{2Z}{c}\frac{n}{60}p\Phi \qquad T = \frac{I_a Z p\Phi}{2\pi c}$ where c = 2 (wave) or 2p (lap)

$V = E \pm I_a R_a$



Shunt motor

Series motor AC or DC

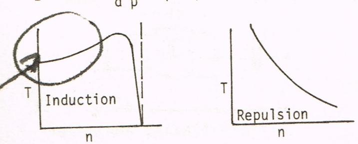
Compound motor

AC Machines

Synchronous speed $= f/p$

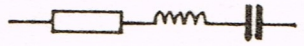
$E = 2.22\,k_d k_p Z f\Phi(\mathrm{rms})$

$T \propto \frac{\Phi^2\,sR}{R^2+(sX_o)^2}$



unstable.

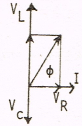
AC Circuits $V_{rms} = \frac{1}{\sqrt{2}} V_{max}$

Series LCR 

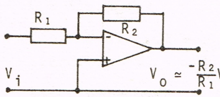
$$Z = (R^2 + (\omega L - \frac{1}{\omega C})^2)^{\frac{1}{2}} \qquad \omega = 2\pi f$$

$$\underline{Z} = R + j\omega L + \frac{1}{J\omega C} \qquad \cos\phi = \frac{R}{Z}$$

At resonance $\omega = \omega_o = \frac{1}{\sqrt{LC}}$ Q factor $= \omega_o \frac{L}{R}$

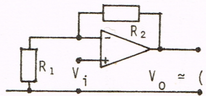


Basic Op'Amp' Circuits



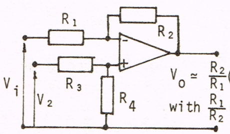
$V_o \simeq -\frac{R_2}{R_1} V_i$

inverting amplifier



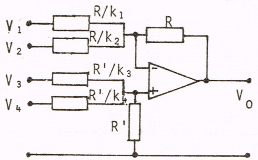
$V_o \simeq (1 + \frac{R_2}{R_1}) V_i$

non-inverting amplifier



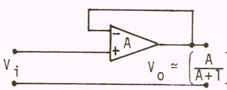
$V_o \simeq \frac{R_2}{R_1}(V_2 - V_1)$ with $\frac{R_1}{R_2} = \frac{R_3}{R_4}$

Differential input amplifier



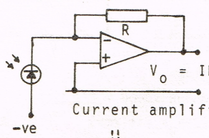
$V_o \simeq k_3 V_3 + k_4 V_4 - (k_1 V_1 + k_2 V_2)$

Adding/Subtracting/Scaling



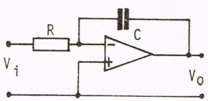
$V_o \simeq \left(\frac{A}{A+1}\right) V_i$

Voltage follower



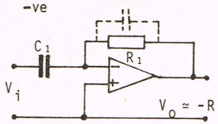
$V_o = IR$

Current amplifier



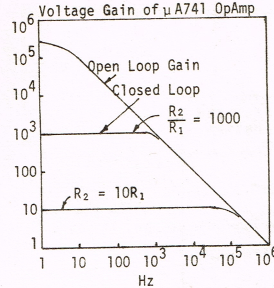
$V_o \simeq \frac{-1}{RC}\int_0^t V_i\,dt$

Integrating amplifier



$V_o \simeq -R_1 C_1 \frac{dV_i}{dt}$

Differentiating Amplifier



Colour Code

0 Black	2 Red	4 Yellow	6 Blue	8 Grey
1 Brown	3 Orange	5 Green	7 Violet	9 White

Preferred Values

10, 12, 15, 18, 22, 27, 33, 39, 47, 56, 68, 82

11 SOIL MECHANICS

11.1 Soil Classification

1. Size Classification

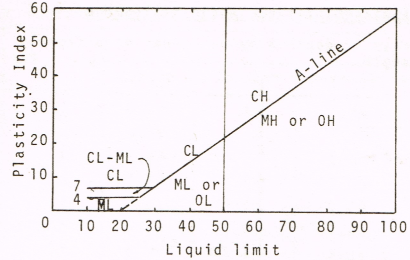
CLASSIFICATION		M.I.T. Size limits mm	B.S. Sieves used for separation mm
Gravel	Coarse		
		20	20
	Medium		
		6	6.3
	Fine		
		2	2
Sand	Coarse		
		0.6	0.6
	Medium		
		0.2	0.212
	Fine		
		0.06	0.063
Silt	Coarse		
		0.02	
	Medium		
		0.006	
	Fine		
		0.002	
Clay			

2. Casagrande Soil Classification, fine grained (50% or more passing B.S. No.200 sieve)

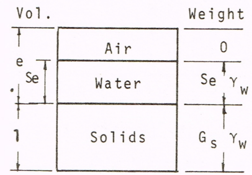
Silts and clays (Liquid limit less than 50)	Inorganic silts, silty or clayey fine sands, with slight plasticity	ML
	Inorganic clays, silty clays, sandy clays of low plasticity	CL
	Organic silts and organic silty clays of low plasticity	OL
Silts and clays (Liquid limit greater than 50)	Inorganic silts of high plasticity	MH
	Inorganic clays of high plasticity	CH
	Organic clays of high plasticity	OH
Highly organic soils	Peat and other highly organic soils	Pt

M silt
C clay
O organic

L low pasticity
H high plasticity



3. Volume-weight Relationships



$$n = \frac{e}{1+e} \qquad e = \frac{n}{1-n}$$

$$w = \frac{Se}{G_s}$$

$$\gamma = \frac{G_s(1+w)}{1+e}\gamma_w$$

$$\gamma' = \frac{G_s - 1}{1+e}\gamma_w = \gamma_{sat} - \gamma_w$$

4. Stratigraphic Table

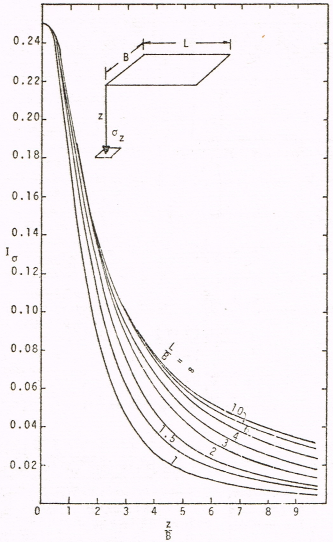
Era	Period	Epoch
Cenozoic	Quaternary	Recent Pleistocene
	Tertiary	Pliocene Miocene Oligocene Eocene Paleocene
Mesozoic	Cretaceous Jurassic Triassic	
Paleozoic	Permian Carboniferous Devonian Silurian Ordovician Cambrian	
Precambrian		

Subdivisions of Quaternary

Relative Climate	U.K. Name
Warm (current)	Flandrian (Holocene)
Cold	Devensian
Warm	Ipswichian
Cold	Wolstonian
Warm	Hoxnian
Cold	Anglian
Warm	Cromerian
Cold	Beestonian
Warm	Pastonian
Cold	Baventian
Warm	Antian
Cold	Thurnian
	Ludhamian
	Waltonian

11.2 Stresses and Displacements in Elastic Half-space

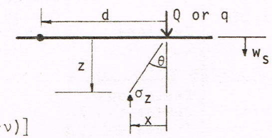
1. Vertical stress at depth z below corner of uniformly loaded rectangle



$$\sigma_z = qI_\sigma$$

2. Boussinesq

(a) Point load Q at surface



$$\sigma_z = \frac{3Q}{2\pi z^2}\cos^5\theta$$

$$w_s = \frac{Q(1+\nu)}{2\pi zE}\cos\theta\left[\cos^2\theta+2(1-\nu)\right]$$

(b) Line load q at surface

$$\sigma_z = \frac{2q}{\pi z}\cos^4\theta$$

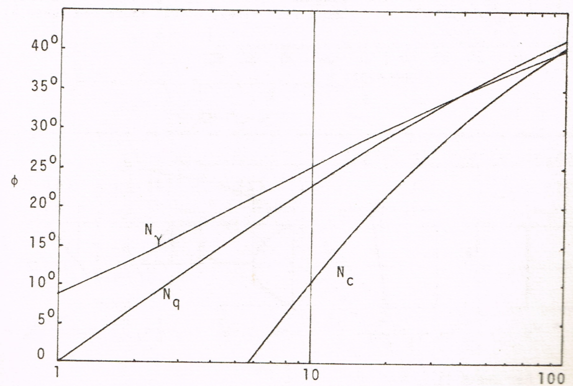
$$w_s = \frac{2q(1-\nu^2)}{\pi E}\ell n\left(\frac{d}{x}\right)$$ where displacement at d is assumed = 0 $(d \geqslant x)$

3. Surface displacement of uniformly loaded rectangle

L/B Ratio	I_δ		
	Centre	Corner	Average
1	1.12	.56	.95
1.5	1.36	.68	1.15
2	1.53	.76	1.30
3	1.78	.89	1.53
4	1.96	.98	1.70
5	2.10	1.05	1.83
7	2.33	1.16	2.04
10	2.53	1.27	2.25
20	2.95	1.47	2.64
30	3.23	1.61	2.88
50	3.54	1.77	3.22
100	4.01	2.00	3.69
Circle	1.00	Edge .64	.85

$$w_s = qB\,\frac{1-\nu^2}{E}\,I_\delta$$

11.3 Terzaghi Bearing Capacity Factors



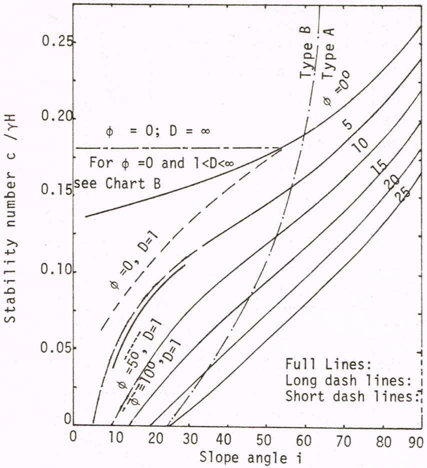
Strip footing $q = cN_c + \gamma DN_q + .5\gamma\ BN_\gamma$
Square footing $q = 1.3\ cN_c + \gamma DN_q + .4\gamma\ BN_\gamma$
Circular footing $q = 1.3\ cN_c + \gamma DN_q + .3\gamma\ BN_\gamma$

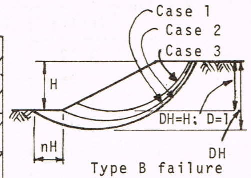
B = FOOTING WIDTH

Note: Reduce c and $\tan\phi$ to two thirds of measured values for local shear

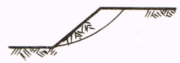
11.4 Slope Stability

TAYLOR STABILITY NUMBERS : CHART A





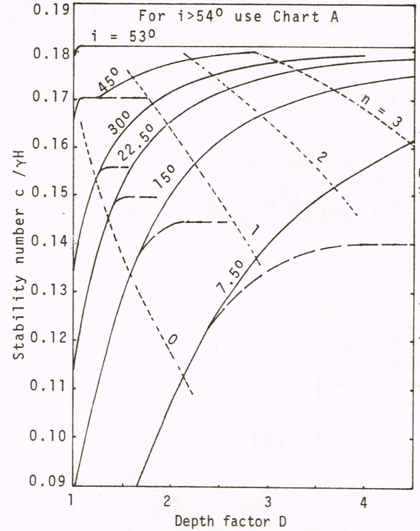
Type B failure

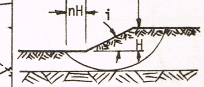


Type A failure

11.4 (cont)

TAYLOR STABILITY NUMBERS : CHART B (purely cohesive soil of limited depth)

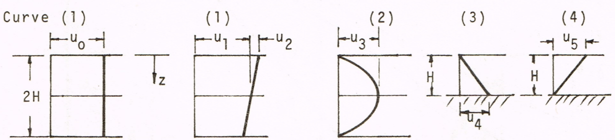




Case A. Use full lines of chart; short dashed lines give n values.

Case B. Use long dashed lines of chart

11.5 Consolidation-Time Curves



$u_i = u_o$ $\quad$ $u_i = u_1 + u_2 \frac{H-z}{H}$ $\quad$ $u_i = u_3 \sin\frac{\pi z}{2H}$ $\quad$ $u_i = \frac{z}{H} u_4$ $\quad$ $u_i = \frac{H-z}{H} u_5$

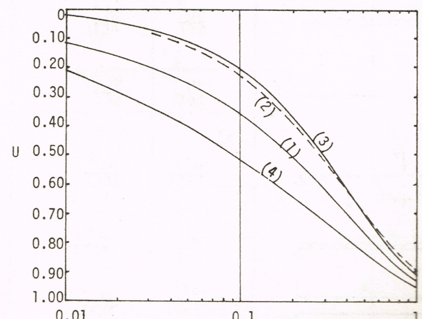
For curve (1): $U < .60 \quad T = \frac{\pi}{4} U^2$

$U > .60 \quad T = -.933 \log_{10}(1-U) - 0.085$

$T_{50} = .197$

$T_{90} = .848$

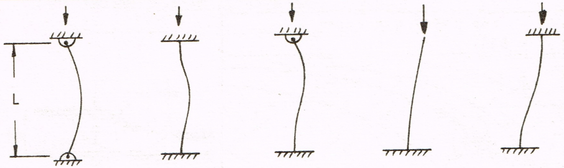
11.5 (cont)



$T = \frac{c_v t}{H^2}$

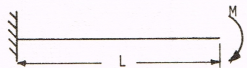
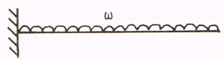
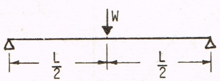
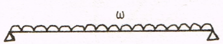
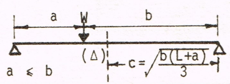
12. STRUCTURES

Buckling Loads

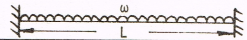
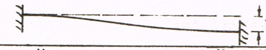
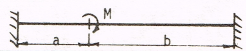
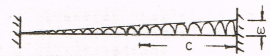
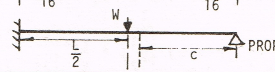
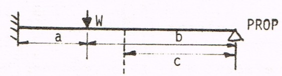
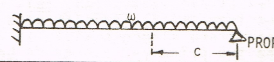


Buckling Load:	$\frac{\pi^2 EI}{L^2}$	$\frac{4\pi^2 EI}{L^2}$	$\frac{2.045\pi^2 EI}{L^2}$	$\frac{\pi^2 EI}{4L^2}$	$\frac{\pi^2 EI}{L^2}$
Effective Length:	L	0.5L	0.699L	2L	L

Beams bent about principal axis

ω is load/unit length	end slope	maximum deflection Δ
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$
	$\frac{\omega L^3}{6EI}$	$\frac{\omega L^4}{8EI}$
	$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$
	$\frac{\omega L^3}{24EI}$	$\frac{5\omega L^4}{384EI}$
	$\theta_b = \frac{Wac^2}{2LEI}$ $\theta_a = \frac{L+b}{L+a}\cdot\theta_b$	$\frac{Wac^3}{3LEI}$ a ≤ b

Fixed End Moments

LH end conditions moment	shear	RH end conditions shear	moment	maximum deflection Δ	maximum deflection position c
$\frac{\omega L^2}{12}$	$\frac{\omega L}{2}$	$\frac{\omega L}{2}$	$\frac{\omega L^2}{12}$	$\frac{\omega L^4}{384EI}$	$\frac{L}{2}$
					
$\frac{WL}{8}$	$\frac{W}{2}$	$\frac{W}{2}$	$\frac{WL}{8}$	$\frac{WL^3}{192EI}$	$\frac{L}{2}$
					
$\frac{Wab^2}{L^2}$	$\frac{Wb^2(L+2a)}{L^3}$	$\frac{Wa^2(L+2b)}{L^3}$	$\frac{Wa^2b}{L^2}$	$\frac{2Wa^2b^3}{3EI(L+2b)^2}$ a ≤ b	$\frac{2Lb}{L+2b}$
					
$\frac{6EI}{L^2}$	$\frac{12EI}{L^3}$	$\frac{12EI}{L^3}$	$\frac{6EI}{L^2}$	1	-
					
$\frac{Mb}{L^2}(2a-b)$	$\frac{M}{L}$	$\frac{M}{L}$	$\frac{Ma}{L^2}(2b-a)$	-	-
					
$\frac{\omega L^2}{30}$	$\frac{3\omega L}{20}$	$\frac{7\omega L}{20}$	$\frac{\omega L^2}{20}$	$\frac{\omega L^4}{764EI}$	0.475L
					
$\frac{3WL}{16}$	$\frac{11W}{16}$	$\frac{5W}{16}$	0	$\frac{2WL^3}{215EI}$	0.447L
					
$\frac{Wab(L+b)}{2L^2}$	$\frac{Wb}{L}+\frac{M}{L}$	$\frac{Wa}{L}-\frac{M}{L}$	0	$\frac{Wa^2b}{6EI}\sqrt{\frac{b}{2L+b}}$ b≥0.4142L	$L\sqrt{\frac{b}{2L+b}}$ b≥0.4142L
					
$\frac{\omega L^2}{8}$	$\frac{5\omega L}{8}$	$\frac{3\omega L}{8}$	0	$\frac{\omega L^4}{185EI}$	0.422L
					

Relations with elastic constants

$G = E/(2(1 + \nu))$ $\qquad K = E/(3(1 - 2\nu))$

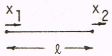
Simple bending $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

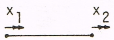
Torsion of circular section $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$

Beam stiffness Coefficients

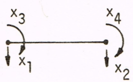
In the following the F's can be axial or shear forces, or, bending or torsional couples corresponding to the mode of deformation.

All beam and frame stiffness matrices may be built up from the following components of each beam element.

(a) axial stiffness  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EA}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

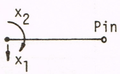
(b) torsional stiffness  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{GJ}{\ell}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(c) bending stiffness and lateral deflection stiffness in one plane

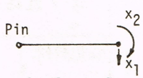


giving $$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \frac{EI}{\ell^3}\begin{bmatrix} 12 & -12 & 6\ell & 6\ell \\ -12 & 12 & -6\ell & -6\ell \\ 6\ell & -6\ell & 4\ell^2 & 2\ell^2 \\ 6\ell & -6\ell & 2\ell^2 & 4\ell^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

In each case if one end is fixed and considered as a reaction. its deflections and forces may be ignored with a corresponding reduction of the stiffness matrix. Another possible form of reaction for case (c) occurs if the reaction end is pinned. Then the stiffness matrix components for the other end are given by

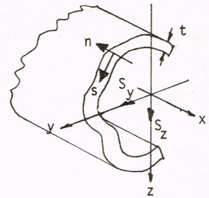
(c)(i)  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EI}{\ell^3}\begin{bmatrix} 3 & 3\ell \\ 3\ell & 3\ell^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

or

(c)(ii)  giving $\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{EI}{\ell^2}\begin{bmatrix} 3 & -3\ell \\ -3\ell & 3\ell^2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

A general plane frame element will have components (a) and (c) and require a 6 x 6 stiffness matrix. A general space frame element will require components of (a), (b) and (c) - the latter twice for two planes of bending - and will require a 12 x 12 stiffness matrix. The three modes of deflection (a), (b), (c) are orthogonal and may be combined into larger matrices with 0's in all unspecified positions. Space frame elements will, in general, have different values of I in the two principal planes of bending.

Shear



Shear flow per unit length of wall resulting from the applied shear forces S_z, S_y is

$$q = t\tau_{xs} = \frac{(-S_z)}{I_{yy}I_{zz}-(I_{yz})^2}\left[I_{yy}\int_A z\,dA - I_{yz}\int_A y\,dA\right] + \frac{(-S_y)}{I_{yy}I_{zz}-(I_{yz})^2}\left[I_{zz}\int_A y\,dA - I_{yz}\int_A z\,dA\right]$$

The resultant force from this shear flow acts through the SHEAR CENTRE.

Torsion

For a circular section $\frac{T_x}{J} = \frac{\tau_{\theta x}}{r} = \frac{G\theta}{L}$

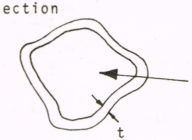
$$J = \frac{\pi D^4}{32} \quad \text{for a solid section}$$

$$= \frac{\pi}{32}\left[D^4_{outer} - D^4_{inner}\right] \quad \text{for a hollow section}$$

For a thin walled closed section

$$T_x = 2Aq$$

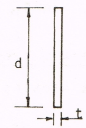
$$= \frac{4A^2 G}{\int_0^s \frac{ds}{t}} \cdot \frac{\theta}{L}$$



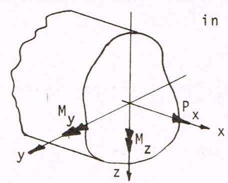
For a thin rectangular section

$$T_x = \frac{dt^2}{3}\tau_{zx\,max}$$

$$= \frac{dt^3 G}{3}\frac{\theta}{L}$$



Asymmetric Bending



in terms of general axes

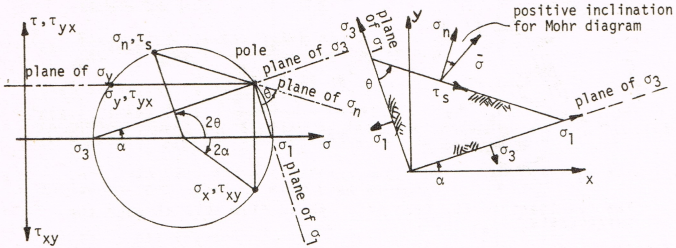
$$\sigma_{xx} = \frac{P_x}{A} + \frac{M_y(zI_{zz}-yI_{yz})}{I_{yy}I_{zz}-(I_{yz})^2} - \frac{M_z(yI_{yy}-zI_{yz})}{I_{yy}I_{zz}-(I_{yz})^2}$$

When the principal axes m,n lie in directions y,z, then:

$$\sigma_{xx} = \frac{P}{A} + \frac{nM_m}{I_{mm}} - \frac{mM_n}{I_{nn}}$$

Stress and strain transformations

Mohr circle of stresses



Equilibrium of prism in σ_1, σ_3 directions gives

$$\sigma_1 \cos\theta = \sigma_n \cos\theta + \tau_s \sin\theta$$
$$\sigma_3 \sin\theta = \sigma_n \sin\theta - \tau_s \cos\theta$$

Then: $\sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta = \sigma_n = \frac{\sigma_1}{2}(1+\cos 2\theta) + \frac{\sigma_3}{2}(1-\cos 2\theta)$

$$= \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2} \cos 2\theta$$

$$(\sigma_1-\sigma_3)\sin\theta\cos\theta = \tau_s = \frac{\sigma_1-\sigma_3}{2} \sin 2\theta$$

Also: $\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \tau_{max}$; $\sigma_3 = \frac{\sigma_x+\sigma_y}{2} - \tau_{max}$; $\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}$

and: $\tan 2\alpha = \frac{-2\tau_{xy}}{\sigma_x-\sigma_y}$

Two-dimensional strain system

$\varepsilon_x, \varepsilon_y, \varepsilon$ are direct strains 'corresponding' to $\sigma_x, \sigma_y, \sigma$

$\frac{\gamma_{xy}}{2}, \frac{\gamma}{2}$ are shear strains 'corresponding' to τ_{xy}, τ

Three-dimensional stress system

If the principal stresses are σ_1, σ_2, σ_3, the principal shear stresses are $(\sigma_1 - \sigma_2)/2$, $(\sigma_2 - \sigma_3)/2$ and $(\sigma_3 - \sigma_1)/2$.

Strain energy per unit volume U may be expressed as

$$U = (\sigma_1 + \sigma_2 + \sigma_3)^2/18K$$
$$+\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]/12G$$

13. SYMBOLS INDEX

GREEK ALPHABET

Α, α alpha	Η, η eta	Ν, ν nu	Τ, τ tau
Β, β beta	Θ, θ theta	Ξ, ξ xi	Υ, υ upsilon
Γ, γ gamma	Ι, ι iota	Ο, ο omicron	Φ, φ phi
Δ, δ delta	Κ, κ kappa	Π, π pi	Χ, χ chi
Ε, ε epsilon	Λ, λ lambda	Ρ, ρ rho	Ψ, ψ psi
Ζ, ζ zeta	Μ, μ mu	Σ, σ sigma	Ω, ω omega

MATHEMATICAL SYMBOLS

- L[] - Laplace Transform
- $\underline{\Delta}$ - defined as
- Σ - repeated summation
- Π - repeated multiplication
- ∂ - partial differential
- |a| - modulus
- ∇ - Laplace differential, Del, Nabla
- $\underline{a}$ - vector
- $\hat{\underline{a}}$ - unit vector
- ⊥ - 'at right angles to'
- . - scalar (dot) product
- x - vector (cross) product
- Re - real part of complex number
- Im - imaginary part of complex number

Symbol	Page No. of use		Recommended S.I. Unit
a	51	velocity of sound	m/s
a	38	lattice parameter	m
a	41	½ crack length	m
a	60	area	m^2
a	37	acceleration	m/s^2
A	48,50	area	m^2
A	38	Atomic weight	-
A	47	availability function (non-flow)	kJ
B	47	availability function (flow)	kJ
B	60	magnetic flux density	T
B	66,67	breadth of footing	m

* of specified fluid

c	67	cohesion	kN/m^2
c	49	velocity	m/s
c_p, c_v	47	specific heat	kJ/(kgK)
c_v	69	coefficient of consolidation	m^2/s
C	60	capacitance	F
C	39	concentration	mol/m^2
C	50	Chézy coefficient	$(m^{\frac{1}{2}})/s$
C_d	50,51	discharge coefficient	-
C_d	51	drag coefficient	-
d	38	interatomic spacing	m
d( )	38	interplanar spacing	m
d	50	depth of flow	m
d_1, d_2	50	" " " before jump, after jump	m
D	51,54	diameter	m
D	39	diffusion coefficient	m^2/s
D	67	depth of overburden	m
D	68,69	depth factor	-
e	65	void ratio	-
E	41,70	Young's Modulus	N/m^2
ΔE	38	energy difference	J
f	61	supply frequency	Hz
f	54	friction factor	-
f	51	wall shear stress coefficient	-
f	47	specific Helmholtz free energy function	kJ/kg
F	47	Helmholtz free energy function	kJ
F, F_1	50,51	Froude Number, before jump	-
F	60	force	N
g	47	specific Gibbs free energy function	kJ/kg
G_s	65	specific gravity of solids	-
G	47	Gibbs free energy function	kJ
G	41,72	modulus of rigidity	N/m^2
G_1, G_2, G	55	transfer function	-
h_f	51,54	frictional head loss	m*
h	47	specific enthalpy	kJ/kg
h	51	heat transfer coefficient	$W/(m^2K)$
H	47	enthalpy	kJ
H	60	magnetising force magnetic field strength	A/m
H	55	transfer function	-
I	60	current	A
I	35,70	second moment of area	m^4
I	35	moment of inertia	kgm^2
J	39	diffusion flux	$kg/(m^2s)$
k	48,51	thermal conductivity	W/(mK)
k	54	surface roughness	µm
k	35,49	radius of gyration	m
k_d, k_p	61	distribution factor, pitch factor of winding	-
K	55	gain constant	-
K	41	stress intensity factor	$MNm^{-3/2}$
K	41,72	bulk modulus	N/m^2
ΔK	41	stress intensity range	$MNm^{-3/2}$
K_p	47	equilibrium constant $(atmos)^{1/n}$	
ℓ	51	length	m
ℓ	60	length of conductor	m
L	51	characteristic length	m
L	60	inductance	H
m	47	mass	kg
m	51,54	mean hydraulic radius	m
$\dot{m}$	47,50	mass flowrate	kg/s

M	70	moment	Nm
M	51	Mach number	-
n	65	porosity	-
n	61	speed of rotation	r/min
n	50	Manning roughness coefficient	-
n	38	atoms per unit cell	-
n_i	41	total cycles at stress amplitude	-
N_p	41	total cycles to failure at strain amplitude	-
N_i	41	" " " " stress "	-
N	60,61	number of turns	-
N	7,38	Avogadro's number	kg/(kgmol)
N	47	cycles per unit time	Hz
N_c	39	total number of atoms	-
p	30	probability	-
p_m	47	mean effective pressure	N/m^2
p	61	number of pole pairs	-
P	47	thermodynamic probability or No.of quantum states	-
P_i	47	engine indicated power	W
q	66,67	surface normal stress	kN/m^2
q	48	heat flowrate per unit area emissive power	W/m^2
q_b	48	emissive power for black body	W/m^2
Q	39	activation energy	J
Q	47	heat (input + ve)	kJ
Q	49	volumetric flowrate	m^3/s
Q	41	crack shape factor	-
Q	60	charge	C
r	41	r.m.s. of stress	MN/m^2
R	60,61	resistance, resistance per phase	Ω
R	50	hydraulic radius	m
R	47	characteristic gas constant	kJ/(kgK)
R_o	47	Universal gas constant	kJ/(kgmolK)
s	47	specific entropy	kJ/(kgK)
s	61	fractional slip	-
S	65	degree of saturation	-
S	47	entropy	kJ/K
S	50	channel slope in uniform flow	-
S_f	50	friction slope	-
S_o	50	invert slope	-
t	37	time	s
T	68	time factor	-
T	61	torque	Nm
T	50	water surface width	m
T	47	temperature (absolute)	K
ΔT	48	temperature difference	°C
T_g	43	glass transition temperature	K
u	47	specific internal energy	kJ/kg
u	37	velocity	m/s
u	68	pore pressure	kN/m^2
U	68	degree of consolidation	-
U	47	internal energy	kJ
v	47	specific volume	m^3/kg
v	37	velocity	m/s
v_o	47	molar volume	$m^3/(kgmol)$
V	60	voltage	V
V	38,49	volume	m^3
V	38	volume of unit cell	m^3
V	50,51	velocity	m/s
V_s	47	cylinder swept volume	m^3

w	65	water content	-
w_s	66,67	surface displacement	m
W	47	work (output,+ve)	kJ
X_o	61	leakage reactance per phase	Ω
Y	49	change in specific energy through a machine	J/kg
z	49	potential head	m*
Z	61	number of armature conductors	-
Z	62	impedance	Ω
α	41	coefficient of linear expansion	μm/(mK)
α	60	resistance coefficient	Ω/K
β	51	coefficient of volumetric expansion	K^{-1}
γ	65	unit weight of soil	kN/m^3
γ	47,50	specific heat ratio	-
γ_w	65	unit weight of water	kN/m^3
γ_{sat}	65	unit weight of saturated soil	kN/m^3
γ'	65	submerged unit weight of soil	kN/m^3
ε	54	relative roughness	-
$\varepsilon_o, \varepsilon_r$	60	permittivity, free space, relative	F/m,-
ε	48	emissivity	-
ε_p	41	plastic straining range	μm/m
η	49	viscosity (dynamic)	mNs/m^2
μ_o, μ_r	60	permeability of free space, relative	H/m, -
μ	49,51	dynamic viscosity	mNs/m^2
μ	37	coefficient of friction	-
ν	72,41,66,67	Poisson's ratio	-
ξ	58,23	damping ratio	-
ρ_o	60	resistivity	Ωm
ρ	41,42,43,51	density	kg/m^3
σ	48	Stefan-Boltzmann constant	$W/(m^2K^4)$
σ	51	surface tension in contact with air	N/m
σ_y	41	proof or yield stress	N/m^2
σ_f	41	ultimate (failure) stress	N/m^2
τ	49,51	shear stress	N/m^2
ϕ	67	friction angle of soil	°
Φ	60,61	magnetic flux, flux per pole	Wb
ω	70,71	load per unit length	N/m
ω	51	angular velocity	rad/s
ω_n	58	natural frequency	rad/s
ω_o	23,62	natural frequency	rad/s
ω_d	59	damped natural frequency	rad/s

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ISBN 0-333-25829-0

9 780333 258293

An Engineering Data Book

A J Munday and R A Farrar

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An Engineering Data Book

Edited by

A J Munday and R A Farrar